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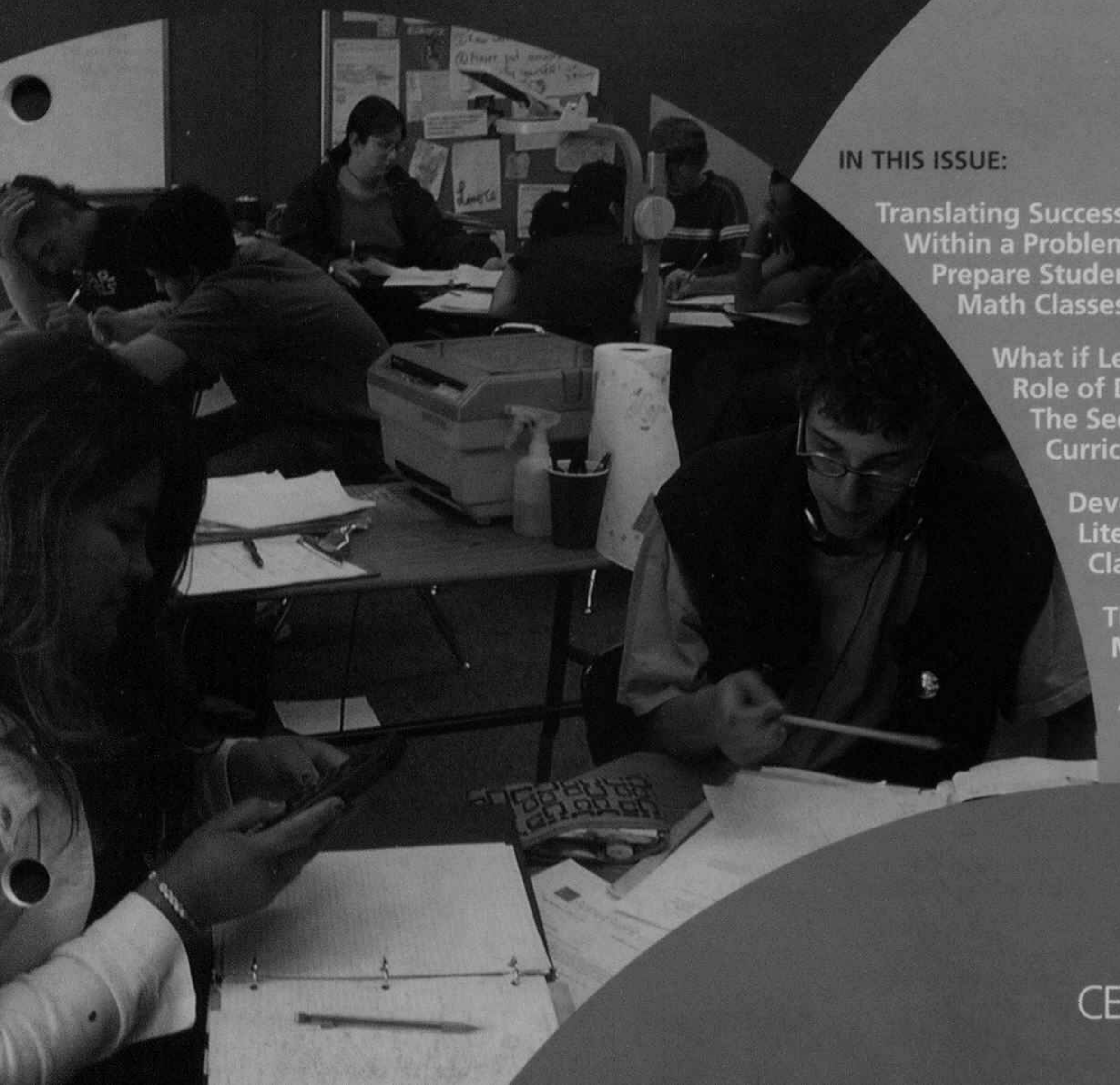

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CESPublications

Testimonials

One year after mathematics coursework at North Central Charter Essential School, structured as described above, students were asked to share their ideas around the principle, "less is more." Some of their responses:

"Rather than just knowing what to move around, or what to plug into where, looking deeper into a topic helped me understand, and I still remember how things worked and why. An example would be the parabola project I worked on with Nick and Kendall, actually making a parabola, and seeing how it should work made the whole concept very clear to understand." -Meaghan Morrissey, current student

"I still find myself remembering the things we covered over the past two years...because of how long we spent on the topics and the projects we did on them." -Chris Foster, current student

"The concept of "less is more" helps me understand a concept, because it allows for more time to study a topic and understand exactly why what happens works. This knowledge in turn made it easier to learn later concepts and processes, for if you know a prior yet related concept the newer one is understood that much easier." -Durrand Michalewicz, current student

Calculus at NCCES "made my calculus class here that much easier, the first part of the course was pretty easy anyway, but once we started getting into real calculus it seemed much more of a review than I thought it was going to be. I realized we'd covered more with you than I'd thought...especially since my roommate was in calc 2 and I'd seen some of what they were doing...I think the best thing about your math class was that it was more traditional than I'd seen before at charter schools (Parker and NCCES) but still non-traditional enough to incorporate the essential school philosophy and allow us to do fun activities/generally have fun." -Kristin Harrington, student at St. Lawrence University

"The calculus class really prepared me for college. I tested out of calculus one at MCLA, it's crazy! And the roller coaster project was really visual, which helped a lot." -Meghan Ekwall, student at Massachusetts College of Liberal Arts

teacher- created school-wide structures that transcended particular disciplines. School-wide Habits of Mind (perspective, evidence, relevance, reflection, connection, and supposition) along with consistent assessment tools provided the needed support for the mathematics program to take root. Each course was guided by a unique essential question (E.Q.) that reflected mathematical processes that teachers and students valued; although general in nature, these questions provided a map for understandings teachers wanted their students to develop (e.g., 10th grade E.Q.: "How do relationships provide evidence to justify conclusions?") The design of each course included outlines of quantitative skills and relevant topics that would help students respond to the E.Q. Direct instruction, routine problems, and more extensive activities and projects constituted classroom activity, generating the tools students needed to explore three mathematical processes through the school-wide Habits of Mind.

The three processes we deemed essential, *mathematical modeling*, *mathematical proof*, and *problem solving*, gave shape and direction to the larger-scope projects that would be portfolio-eligible. Mathematical modeling guided students in finding explanations for relevant phenomena by simplifying a real-world situation using mathematical representations. Through this work, students tested their ideas, determined limitations of their model, and extracted useful results that could inform the original problem. The language of the school-wide habits played a central role in analyzing their models (e.g., supposition: What might happen if we make a small change in one of the variables?).

The *proof* strand consisted of providing students opportunities to justify their ideas using formal mathematical language. Although the design of possible 'proof' portfolio pieces required more teacher guidance, students learned how to make claims from observed patterns and logically organize information to determine the truth of each claim; students pieced together a valid argument from internalized mathematical knowledge.

Finally, the problem-solving strand complemented the other two as it was more directed than a modeling piece, yet still left room for student creativity and exploration of various approaches. These problems helped students move from pattern recognition and testing particular cases to generalization. The Habits of Mind continued to support student learning, helping students make connections from one problem to another or extend ideas to more complex situations.

North Central Charter Essential School: Learning Levels

The mathematics curriculum structure at NCCES reflected a progression of the work begun at New Mission, with the three mathematical processes described above guiding the framework. Giné, the math team leader from New Mission, brought the framework in use for further development at the young Essential school in Fitchburg. In turn, the mathematics team at NCCES, all new to the school that year, had the opportunity and challenge of implementing this framework while further developing it within a different context. Rather than using Habits of Mind to guide classroom activity and student progress, Learning Levels were written to trace possible learning paths for students during their six-year experience at the school (the school serves grades 7 through 12). The Learning Levels would provide consistent language for teachers designing their courses and eventually for student use in identifying their own meta-cognitive processes. Originally developed by the school leaders, Peter Garbus and Melanie Gallo, and by founding teachers, such a progression was already in existence for all disciplines but needed revision within the area of mathematics.

The math faculty created eight categories of mathematical processes we deemed necessary in attaining quantitative literacy as defined above: visualizing, working with graphs, measuring, estimating, using notation, formulating conjectures, proving, and modeling. These processes were applied in the development of learning tasks based on *mathematical modeling*, *problem solving*, and *deductive/inductive reasoning*. Thus, the three revised curricular elements inherited from New Mission were used to describe actual learning activity, such as class problems, activities, and projects through which students applied concepts and skills at an appropriate learning level. The courses were also designed to help students progress at their own pace in each learning level area. We created five levels of Integrated Math classes with guiding essential questions and course content used as vehicles for development of essential understandings reflected in the learning levels and in the three greater mathematical goals. Although in theory students needed to meet Learning Level 4 expectations in order to graduate, we offered a Statistics course and a Calculus course guided by Learning Level 5 to help students understanding extend further.

Parker School Criteria for Excellence

Problem-Solving

- You understand the problem.
- You identify special factors that influence your approach before you start.
- Your approach is efficient or sophisticated.
- You clearly explain the reasons for your decisions along the way.
- You solve the problem and make a general rule about the solution.
- You extend what you find to a more complicated situation.

Communication

- You use appropriate mathematical language to communicate your solution.
- You use graphs, tables, charts, and/or drawings to communicate your solution.
- Your work is well organized and detailed.

Several learning tasks within this unit were developed and used at New Mission and NCCES

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Do I/ Does the Student...

Level 1

- Actively explore how one, two, and three dimensional shapes interact (e.g., investigate volume by fitting contents of one object into others; submerge irregularly shaped objects in water and measure displacement)
- Understand relative size: compare lengths, areas, and volumes
- Use appropriate units in respective dimensions

Level 2

Same as Level 1 and:

- Understand inherent relationships within the same object (e.g., Pythagorean Theorem for right triangles)
- *Understand relationships* among properties of objects (e.g., discover how many cones fit within a cylinder with the same height and base)
- Measure angles

Level 3

Same as Level 2 and:

- Represent measurements of objects using equations
- Solve problems and explore applications using formal equations
- Measure unknown quantities indirectly (e.g., using triangles & similarity)
- Apply understanding of length, area, volume, etc. to real-world problems
- Use significant digits when calculating error in measurement

Level 4

Same as Level 3 and:

- Justify mathematical expressions of measurement (e.g., formulas for volume of cone, area of triangle)
- Measure indirectly (e.g., using trigonometry)

Level 5

Same as Level 4 and:

- Make a conjecture based on observations and use a logical argument to prove it

Above is a sample of the progression outline for "measuring," one of our eight learning levels for mathematical reasoning.

Francis W. Parker Charter Essential School: Criteria for Excellence

Authors Diane Kruse and Roser Giné currently teach at Parker. The mathematics curriculum at Parker is embedded in the six-year integrated Math, Science, and Technology program. During the first four years (Divisions 1 and 2), students all experience the same core curriculum in two-hour classes team-taught by a math and science teacher. In the final years of the program (Division 3), students take separate one-hour courses selected from a range of courses that allow them to make some choices based on their future goals. Approximately half of all graduates take calculus each year.

The organizing conceptual framework for Parker's math program is deceptively simple: as they progress through the Divisions, students indicate their readiness to move from one level to the next by demonstrating increased sophistication in the two areas of *Mathematical Problem Solving* and *Mathematical Communication*. Throughout the program, students demonstrate their ability to meet standards in these areas through their performance on messy, open-ended tasks that require creative thinking, application of concepts explored in class, and clear communication of the process involved in solving the problem.

In Division 1, Parker's middle school students tackle regular Challenges of the Week (COWs) that relate to both the math and the science content being explored in class. As problem solvers, students at this level are learning to make connections between the disciplines, and to be persistent when a problem takes more than one day to solve. Classroom activities and instruction are designed to foster a spirit of inquiry, as well as to provide practice with some of the underlying skills and content that students are learning. Over the course of two years, students improve their ability to organize data in several forms and to find patterns and trends in that data that tell a story. They begin using algebra, diagrams and other strategic approaches to organize what they know and figure out what to try next. They start to develop the habit of finding more than a one-solution approach to a problem. The emphasis on communication at this level is primarily on fully explaining the solution process, showing all work, and answering all questions fully and completely. While they are learning some of the conventions for formal mathematical communication (particularly the use of graphs, charts, and tables), students at this level may still be quite wordy in their discussion of a problem, since their thinking is more concrete and the emphasis is on getting all of what is in their heads down on paper.

In Division 2, students formalize their study of algebra and geometry and learn more techniques for data analysis in the context of their science work. Assessments are more varied and may include problems of the week, major projects, and in-class academic prompts. Problem solving in Division 2 demands a greater level of algebraic thinking and abstraction, and students are expected to use multiple approaches to verify their solutions to problems. Mathematical communication becomes more formal as well, as students start paring their wordy discussions into more efficient symbolic explanations, and shift their tone from first to third person. In particular, students develop a deeper understanding of the use of variables, both for problem solving and for effective communication.

By Division 3, students are ready for a great deal of abstraction. They are expected to approach any mathematical problem solving task with a clear and systematic approach, where they frame and organize what they know, make connections to content and techniques that may prove useful, carry out a solution to the problem, then verify their work, sometimes with formal proofs. Communication at this level is highly technical, using all of the conventions of the discipline to be clear, concise, and efficient. Students edit, revise, and proofread their work to ensure the appropriate level of formality.

What does this look like in practice?

Division 2: Disease Unit

The Disease Unit at Parker consists of an eight-week learning experience guided by three essential questions:*

How can we quantify non-constant change?

How can we use mathematical models to gain information about a particular phenomenon?

How can we model the spread of an epidemic? Why is this useful?

The initial generative task was to investigate the interaction between sickle cell anemia and malaria, using a two-week whole class investigation (source: "A Study of Sickle Cell Anemia: A Hands-On Mathematical Investigation," by Rosalie Dance and James Sandefur, 1998; project supported by the National Science Foundation). This class activity introduced students to non-constant change, exposing them to functions beyond linear and forming a bridge into exponential.

* Several learning tasks within this unit were developed and used at New Mission and NCCES

Students simulated births from a parent population with a given proportion of normal alleles and mutant alleles (sickle cell) in an environment where malaria is a risk; determining conditions were also provided, yielding distinct proportions of sickle cell and malaria survivors.

The initial goal for the class was to find a function that models the situation described using introductory probability theory. Subsequently, students searched for an input value that could maximize the total number of survivors.

We found this activity to be rich with essential mathematical ideas that would exercise students' ability to construct a math model from a realistic situation and would yield many possible natural connections to a Humanities curriculum. Classes at NCCES and at Parker engaged in this work, and students found different entry points given their individual cognitive skills, while coming together through classroom activity. This served the populations of both schools well, as each had heterogeneously grouped classes. Thus, some students who had experience with quadratic functions applied their function notation skills and their algebraic skills to generate the quadratic equation and explore changes in initial conditions, while first-year students used technology to inform their models. All students were able to experience aspects of probability theory and connections to genetic diseases, both topics of study receiving an in-depth focus during the next curricular year.

Through activities, direct instruction, collaborative work, and oral presentations, the rest of the unit facilitated student development of algebraic skills particular to exponential functions while applying modeling processes to different situations. The final part of the unit focused solely on modeling and introduced students to regression and to methods used to determine the predictive value of generated models (i.e., residuals, correlation coefficient, residual plots).

The culminating learning task and assessment was modified from a similar task initially developed at New Mission, with changes implemented at NCCES. This involved modeling the growth of an epidemic, interpolating or extrapolating from the data using best-fit curves, and analyzing error from regression. Finally, students used mathematical language to communicate their findings, either through a structured report or through a news story set in the time of the disease's greatest impact.

The disease unit combined routine problems mixed with directed problem solving to support student exploration of messy problems in more realistic settings. Although the time spent on this unit was significant, students walked away from the experience with a clearer sense of the power and practice of mathematics.

Division 3: Trigonometry and Geodesic Domes

The Geodesic Dome project has become an annual event in Parker's spring semester trigonometry course. After learning the foundations of right triangle trigonometry and connecting that knowledge to the unit circle and the trigonometric functions, students wrap up the semester by examining what happens when we try to apply trigonometric relationships to non-right triangles.

This unit is a critical example of one of the ways that "less is more" plays out in Parker's program. The basic new content of the unit, the Law of Sines and the Law of Cosines, can be derived and demonstrated in a few brief lessons, and with some practice and application problems, students could be finished and on to new content within a week. However, Parker students spend four weeks designing and constructing geodesic domes, working with the essential question: How can we use right triangle and non-right triangle trigonometric techniques to design and construct a geodesic dome?

To build a geodesic dome, the equilateral triangular faces of a tetrahedron, octahedron, or icosahedron are divided into smaller networks and the vertices of that network are "popped out" to make a rounded figure. For example, a 2V network would find the midpoint of each side of the equilateral triangle (dividing it into two sides), and those midpoints would pop out to form a rounded edge. As students conduct this investigation, they learn about the Platonic solids and prove why there are only three different solids that can be built from equilateral triangle faces. They learn that all of the Platonic solids can be circumscribed, and solve the problem of how the radius of the circumscribing sphere relates to the edge length of each solid. They revisit geometric conventions for naming figures in a diagram, and realize the need for careful naming of each part of their diagrams as the figures quickly become complex (students are visualizing multiple cross-sections of the three-dimensional solid as they attempt to "bump out" different parts of the faces to make a dome). And every step of the way, students are repeatedly searching for triangle relationships – right and non-right – that will allow them to carry out the necessary calculations for building their domes.

The challenge of building a dome is deceptively simple, which allows students to really dig in as problem solvers who need to communicate clearly. As teachers, we can then observe in depth our students' ability to respond effectively to a complex, multifaceted task. We have found that this project appeals to students on different levels. Some students are drawn to the problem solving, while others appreciate having

Continued on next page

physical models in front of them and are motivated to create something interesting or beautiful.

The Dome project has the added benefit of being easy to differentiate. Almost every student at Parker takes trigonometry, including students on special education plans and students who will not take calculus. Students who have a tough time visualizing can build domes with the octahedron as a base, taking advantage of the many familiar right triangles in the form. Students who need more time to complete a project can work with a simpler 2V or 3V network on the triangular face. Students who need a challenge can subdivide the equilateral triangle face into as complex a network as they like (the current record is seven), or work with a more complex design.

Division 3: The Roller Coaster Project

The Roller Coaster Project was implemented at NCCES within the school's first Calculus course and a revised version was used as a first semester Calculus culminating experience at Parker. Both courses, taught through a deductive reasoning approach in which students are exposed to formal proofs when feasible and on occasion are asked to construct their own, were guided by the following essential questions:

1. How can we uncover the concept of 'closeness' using mathematical language?
2. How do we make sense of and quantify non-constant change? What does this allow us to do that we couldn't do without Calculus?
3. What is the connection between definite integrals and the derivative? How is it relevant? ("How is it relevant?" is Parker's school-wide Essential Question for the academic year 2006-2007.)

The project was used to help students refine their understanding of the derivative and its power in optimization problems (source: "Interactive Web-Based Calculus Projects at Hollins University: Area of U.S. States and Roller Coasters", by Julie Clark and Trish Hammer; website: www1.hollins.edu/depts/math/hammer/coaster).

Students used toy coaster models to create their own paths, with the goal of maximizing a determined 'thrill' function based on height of path and angle of steepest descent (for this, both first and second derivatives are applied). The project cited used more sophisticated mathematics technology, yet without access to MAPLE (a highly specialized mathematics software tool), our Calculus classes were nevertheless up to the challenge of creating a coaster and proceeding with analysis of *physical* models. With encouraged peer collaboration, groups of students were given coaster kits and the freedom to design a chosen coaster path given the limitations inherent in the materials (an extension to this project includes minimization of materials or cost as an additional optimization problem). Different groups had varied levels of success with the physical model; getting to the mathematical concepts underlying the problem proved to be a frustrating process for some and a highly engaging process for others. Students who were more comfortable solving problems with one set solution encountered moments of anxiety that pushed them to learn from the strength of others. Simultaneously, those who preferred active learning tasks were challenged to formalize their processes using mathematics by tapping their peers' expertise. Multi-directional learning relationships evolved within a small space saturated with toys.

Creative use of technology also emerged from the collaborative work. The project assignment spawned purposeful use of TI graphing calculators and Geometer's Sketchpad. For instance, some groups uploaded digital photos of their toy coasters and used regression to model the paths. Subsequently, calculus was used in the analysis.

This project asked students to apply the ideas learned through class work, homework, and other learning tasks to a problem that depended on flexible application of the mathematical processes embedded within a first-semester Calculus course. Three weeks at the end of the semester were dedicated to this work, as students constructed their coasters, used technology to find best-fit curves, and applied differentiation techniques to optimize a function. Our sense and experience is that within a more traditional Calculus course, students would have moved faster with the material, leaving review time at the end of the year for an end-of-year exam or for the Advanced Placement test. Although both approaches are indispensable with respect to particular course goals, a project of this scope is valuable because it teaches students how to apply what they've learned in a relatively authentic way, and because it broadens classroom activity. Students take ownership of the work, seeing first-hand that people learn in different ways and most important, experimenting and persevering in a safe environment. NCCES students as well as Parker students are still given the opportunity to take the Advanced Placement test for possible college credit; test preparation is then given additional time, either during school hours, or after school.

Conclusion

In Essential schools, teachers walk a fine line as they attempt to be true to the principle "less is more" within their mathematics programs while ensuring that students are quantitatively literate and prepared for both informed participation in our society and careers within science or mathematical fields. We have found ways to do "less" by organizing content around essential questions, and articulating goals for mathematics instruction that transcend the particular content being studied and instead reflect broader skills and mathematical ways of thinking. This is not without its costs; we know that an observer in our classrooms might see fewer exercise sets and fewer course choices, as well. We worry that our students are less facile with algebraic manipulation and that some routine procedures are less automatic than we would like, and we continue to work on building these component skills into the programs of study we design. It is incumbent upon us to ensure that students obtain needed support in internalizing such routines so that they can become more flexible in concept learning and application. However, students in these programs experience the same critical mathematics *content* as their peers in traditional math programs, and we would argue that they experience this content in ways that engages them deeply and allows them to make more of their mathematics experience.

The kinds of mathematical processes that we have observed in development within our students, along with student products from performance assessments such as the ones described here corroborate our belief that through our programs students are:

1. Transferring mathematical skills and knowledge to non-routine problem situations
2. Developing a meta-cognitive awareness that allows for conscious access of relevant information
3. Internalizing the process of justification
4. Using the language of mathematics to communicate and build upon their ideas.

Although the current standards-based initiative poses a challenge, particularly within schools that have an urgent need to raise test scores, it also provides us with an opportunity to analyze the work we are doing and justify it as we remain accountable to our students.

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Diane Kruse (dkruse@parker.org) is the Domain Leader for Math, Science, and Technology at the Parker School where she is also currently teaching in Divisions 2 and 3. Her twelve years of mathematics teaching have included time in traditional public school environments, alternative schools for students at risk, and an international school in Brussels. Diane is interested in finding ways to make math relevant, interesting, and accessible for all students.

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Discourse Time! Developing Argumentative Literacy in the Math Classroom

by David I. Singer

Mike Schmoker, author of *Results Now*, articulates that "generous amounts of reading, writing, and argument are all essential to the development of truly literate and educated students." Moving this perspective of literacy to the world of mathematics, we may define a numerically literate person as someone who is able to read, write, and argue with numbers, or more important, mathematical concepts and ideas.

Since the start of my career as a math educator, I've been committed to the idea that reading and writing in the math classroom are essential. Cross-content literacy efforts, which are school-wide practices in many cases, have been ingrained in my beliefs around supporting student success. However, such efforts, though meaningful and valuable, have failed to consider the concept of argumentative literacy; the idea that students must be able to share ideas, listen to alternative perspectives, develop counter-arguments and transform their own thinking as well as those around them. Reading and writing comprise two pieces of the literacy puzzle; without argumentative literacy, a person may find him or herself to be illiterate. As Gerald Graff states "Argument literacy is central to being educated. It grants access to forms of intellectual capital that have a lot of power in the world."

The purpose of the following text is to describe Discourse Time (D.T.), a teaching practice that aims to integrate argumentative literacy, the third piece of the literacy puzzle, into our math learning environments. Snapshots from a tenth grade classroom at Skyview Academy High School in Thornton, Colorado will be used to paint a vivid picture of what D.T. looks and feels like in addition to the way it impacts student learning.

Discourse Time!

"We can't argue math, Mr. Singer. It's true, it's specific," insisted Taylor. "Are you sure? I really think we can." "How can you? There's nothing to argue about. There's no side to take," she responded. "Are you positive about that? Do you really think we can't argue mathematics?" Taylor and I continued to debate for a little while until I finally pleaded with her to give me a chance at proving her wrong. "Fair enough," she said, "prove me wrong." With our unresolved argument lingering, I introduced my class to the idea of mathematical discourse.

Having just visited an amazing humanities classroom a day earlier, I took some time to describe what I had seen to my students. "For a half an hour, six kids in the class ran the show. They completely owned the classroom. It was awesome. Mr. Munoz, their teacher, provided them with a few questions to guide their

Skyview Academy High School opened in 2005 as a new small Essential school focused on discussion-based, topic-directed learning for all of the core content areas, project-based learning, and advancement and graduation by portfolios and exhibitions. Currently serving 300 students in grades nine through eleven, Skyview Academy's first senior class will graduate in 2008.

discussion, but other than that, the whole thing was student-centered. They were debating about what kind of person Christopher Columbus was and using all sorts of reference documents, his journals, pictures, history books, whatever else, all in attempts to support their positions. The cool thing was listening to students call each other out. Like, what evidence do you have to back that up? Prove it! You're just talking – you've got no support! Anyway, it was really exciting to watch, and I want to make that learning environment a regular part of our class."

"Aren't you proving my point, Mr. Singer?" chimed in Taylor. "We argue and debate in social studies all the time. That subject's set up for that kind of thing."
"So is math!" Seizing what seemed like a teachable moment, I couldn't help but share my strong opinion on the matter. "Do you really think the rules we use today, the math that we take for granted, was accepted right away? Do you think that other mathematicians just looked over an idea, said okay, sounds good to me, and accepted it as fact? Absolutely not! They argued and debated; they pounded their fists on tables in an effort to emphasize their points. That's what I want you guys to do…pound your fists on tables!"

"You want us to pound our fists on tables?" questioned Omar in a confused manner. "Yes! I want you to get excited about math, I want to give you a chance to really experience what it's like to be mathematicians with one another. If that means pounding your fists on tables, then so be it." And thus the birth of Discourse Time, a once a week ritual that finally invokes the spirit of a quote I've had hanging on my wall for over two years, "You have no right to no opinion."

How does Discourse Time Work?

"Unlike casual conversation…discourse requires a combination of both reflection and action. That is, during the exchange of ideas, participants attempt both to gain insight into the conceptions of others and to influence them" write Azita Manouchehri and Dennis St. John in *Mathematics Teacher*. That's what D.T. is all about: students sharing their ideas, listening to others, and transforming the way they think about mathematics.

CES cofounder Deborah Meier recognizes this belief to its fullest. "Intellectual argument was a daily feature of life at her school, Central Park East Secondary School. For Meier, argument and discussion provide perhaps the best opportunity for kids to become critical thinkers, for us to help them see the power of their ideas," writes Schmoker of her work in *Results Now*. In a democratic school grounded in the ten Common Principles, it's essential for our math classrooms to encourage and support students in sharing their voice.

That's why discourse is so important. Now here's how to make it happen.

Where Should Discourse Happen?

The ideal physical environment for D.T. is a large table around which five to seven students can sit, a dry erase board next to the table for those engaged to present ideas, and a circle of chairs on the outside for the remaining students and teacher to observe.

What Do the Students Directly Involved in the Discourse Do?

The students directly involved in the discourse collaboratively attempt to solve a problem. This isn't the sort of problem that has a clear right or wrong answer, nor is it the kind of problem that students could tackle alone at their desks. It's the kind of problem that forces students to use their minds well. This is the kind of problem that lies directly inside of psychologist Lev Vygotsky's zone of proximal development. As described by Jeff Wilhelm, this learning "zone" is defined as "anything that the child can learn with the assistance and support of a teacher, peers, and the instructional environment." The beauty of D.T. is that it relies heavily on the "peers and instructional environment" components of Vgotsky's zone and far less on the "support of a teacher" component. Our CES principle of "teacher as coach, student as worker" is emphasized in Discourse Time as the ownership of learning is placed heavily on the students while the teacher becomes a facilitator of thinking.

Related Resource

Discourse Time in mathematics bears a clear correspondence to Socratic seminars, a discussion format commonly used in many Essential schools. For more on how Essential schools incorporate Socratic seminars to teach for understanding, read "Less is More: The Secret of Being Essential" in *Horace* Volume 11, Issue 2, November 1994, www.essentialschools.org/cs/resources/view/ces_res/88

In his text *Comprehending Math*, Arthur Hyde makes a succinct and highly enlightening point. "What do I do?" is the right question for kids to ask when attacking a challenging problem in mathematics. However, they've been asking the teacher when all along, they should be asking themselves. D.T. puts this research into practice in the math classroom.

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The problem selected for D.T. should be directly related to the essential learning taking place in class. The experience should act as another opportunity for students to engage in developing their understanding of the formal learner outcomes that have been established. This provides the teacher with another opportunity for formative assessment as she may examine what her students know and are able to do within the essential learning being explored in class.

How Do the Students Sitting at the Table Know What to Do?

Creating a set of expectations that the entire class is both comfortable and familiar with is crucial to the success of Discourse Time. Students participating in D.T. must feel safe in order to take mathematical risks. A scoring system, such as the one outlined below, is a great way of emphasizing what the expectations are and what is truly valued during D.T. Collaboratively creating the scoring system with students in advance of getting involved in a D.T. gets the whole class thinking and discussing what is valued in this activity. Here's how it might look:

- States an opinion relevant to the problem (1 point)
- Draws another person into the discussion (1 point)
- Makes a connection – doesn't have to be math based (1 point)
- Supports an opinion with factual evidence (1 point)
- Asks a question that moves the discussion - pushes the group's thinking (2 points)
- Proves someone else's opinion with evidence (2 points)
- Takes away from the value of D.T. (- 1 point)

> Teaching Note: The purpose of the scoring system is not to work for or against a student's grade. Rather, the goal of the scoring is to emphasize the values of the classroom in a highlighted arena, which in this instance takes the form of Discourse Time. Posting high scores from each class excites students about their contributions to D.T. although some still demand a grade linking their contributions to their overall grade in the course. For help with dealing with those students having a strong desire for their contributions to be linked directly to a grade, please read below in the "What does the teacher do?" section.

While "right and wrong" answers have nothing to do with the scoring system; providing evidence to support your position is a "2-pointer." The scoring system may be revised to meet the needs of any classroom, but should emphasize the core values of the class it is serving. In the end, teachers should examine the agreed-upon scoring system through the lens of the following question, what kind of message does this send to the participants?

What Do the Students Sitting Outside Do?

The students outside of the D.T. can have many different jobs. In the D.T. currently going on in my classroom, there are two roles. The first is to act as a scorer for a student in the middle. If six students are participating in D.T., then there are six scorers sitting on the outside, each assigned to a D.T. participant. Having the scorers sit directly across from the student they're evaluating helps ensure that they may not only hear what students are saying, but also see what students are doing. Simple tally marks documenting when each item on the score sheet is being addressed makes the process fairly easy to follow and also promotes students on the outside to be active observers.

A second job for the remaining outside observers may be to record a double entry journal. One heading focuses on the "I notice…" while the other emphasizes the "I wonder…" For example, a student may write "I notice that Monique doesn't willingly share her ideas" or "I notice that students are easily persuaded to believe Osvaldo's opinions." On the "I wonder" side, you may hear a student state "I wonder why Maria writes so much, but shares so little" or "I wonder who's actually right." These observational notes in the form of a double entry journal do a great job of providing non-participants with an active role. In fact, their notes act as a foundation for the class-wide debrief that may take place directly following the D.T.

What Does the Teacher Do?

As stated earlier, D.T. is a great opportunity for the teacher to check in with her students. The teacher may take notes on what she sees and hears the students doing. These notes may then serve as a formative assessment, guiding her upcoming instructional decisions. They may also act as part of the students' bodies of evidence as they work to prove their learning of the essentials. The notes documented by the teacher may be copied and distributed to the D.T. participants. Examining the teacher generated observation notes, students may highlight the statements they made and use the document as part of their portfolio as a piece of physical evidence toward their level of learning.

Why Do We Need Discourse Time?
Although many classrooms currently experience some form of mathematical discourse in an informal fashion we must take such practice to the next level. The majority of discourse found in our classrooms rarely forces students to take the role of mathematician as they engage in what Ellin Keene defines in *Dimensions of Understanding* as "rigorous discourse." D.T. is merely one way to create an authentic arena for valuable mathematical argument to take place. In addition, setting aside time in our class to address this need sends a message to our students that sharing, defending, and transforming mathematical ideas is an extremely valuable way to develop argumentative literacy; the third component of literacy.

Related Resource

For more on the restructuring of the Mapleton School District of which Skyview Academy High School is a part, read "The Stars Aligned: A Study of System Change in Colorado's Mapleton Public Schools," in *Horace* Volume 21, Issue 3, Summer 2005, www.essentialschools.org/cs/resources/view/ces_res/370

What Does D.T. Actually Look Like When We Do It?
What follows is an example of Discourse Time. The context of the work is taken from a heterogeneous, ninth-grade Algebra/Geometry I course. The sample D.T. includes most elements of the process; due to space constraints, the teacher notes, which are made on a worksheet containing the seven areas in which students score points (listed above) has been omitted. The following example of Discourse Time contains:

- The essential learning being explored in class at the time of D.T.
- The problem used in D.T.
- Student double entry journal notes (I notice… I wonder…)
- A debrief of the experience

Discourse Time Sample 1: The Boat Race
The Essential Learning: Students will use proportional thinking to analyze and solve real world problems within geometric and algebraic contexts.

The Problem: In a two-boat sailing race, one boat, *Windsprite*, rounds the final buoy and sails straight for the finish line at 12.0 knots. Exactly 4 minutes after *Windsprite* rounds the final buoy, the other boat, Porpoise, reaches that point and heads for the finish line at 12.7 knots. *Windsprite* re aches the finish line 49 minutes after rounding the last buoy. Who wins the race? Why?

Student Double Entry Journal Notes

I notice...	**I wonder...**
That most of them are underlining important things	Why Salvador and Monique aren't talking
That some aren't talking	What the actual answer is
That Manuel started to talk and 2 other people started to talk about the same problem	Who knows what the answer is and who is confused
That Singer stepped in to help them with their problem	
Salvador made another connection	
Jessica did a math connection	
Manuel made a math connection	

The Debrief: What made this D.T. challenging for students was their lack of contextual knowledge. Boating is not part of their general schema; as a result they had to develop their background knowledge in order to engage in the work. Recognizing a need to find information prior to jumping into problem solving is a great asset for a mathematician. Moreover, resourcefulness – using resources to accommodate the recognized need – is a skill that all great thinkers possess.

The connections made by participants, both to their work in science class with unit conversions and their own life experiences, were amazing to hear. Those connections bring the problem to life for those students and take it beyond the walls of the isolated math classroom. In addition, the connections also recognized by Daniel in the double entry journal notes let me know that he too witnessed their value. Beyond Daniel's observation about connections, he also recorded "that [teacher] Singer stepped in to help with the problem." During our class debrief, when Daniel mentioned this "I notice" comment to the class, I asked if my involvement was unnecessary or overbearing. The general consensus from the participants was that my involvement pushed the thinking of the participants and never provided solutions or answers. As a result of Daniel's observation, I was able to assess my own involvement in D.T. and its importance or lack thereof to the group effort.

Continued on next page

The Coalition of Essential Schools: Common Principles

Demonstration of mastery

Commitment to the entire school

A tone of decency and trust

Goals apply to all students

Resources dedicated to teaching and learning

Less is more, depth over coverage

Learning to use one's mind well

Personalization

Student-as-worker, teacher-as-coach

Democracy and equity

The Coalition of Essential Schools

Imagine schools where intellectual excitement animates every student's face, teachers work together to improve their craft, and all students thrive and excel. For more than 20 years, the Coalition of Essential Schools (CES) has been at the forefront of making this vision a reality. Guided by a set of Common Principles, CES strives to create and sustain personalized, equitable, and intellectually challenging schools.

The CES network includes hundreds of schools and 25 Affiliate Centers. Diverse in size, population, and programmatic emphasis, Essential schools serve students from kindergarten through high school in urban, suburban, and rural communities.

Essential schools share the Common Principles, a set of beliefs about the purpose and practice of schooling. Reflecting the wisdom of thousands of educators, the 10 Common Principles inspire schools to examine their priorities and design effective structures and instructional practices.

CES was founded in 1984 by Theodore R. Sizer and is headquartered in Oakland, California. Please visit our website at www.essentialschools.org for more about CES's programs, services, and resources.

Horace

CES publishes its journal *Horace* quarterly. Combining research with hands-on resources, *Horace* showcases Essential schools that implement the 10 Common Principles in their structures, practices, and habits. Within four focus areas—school design, classroom practice, leadership, and community connections—*Horace* explores specific questions and challenges that face all schools in the CES network.

Subscriptions to *Horace* are a benefit of affiliating with CES National as a regional center, school, or network friend. We invite you to visit the CES website at www.essentialschools.org for information on affiliation and to read *Horace* issues from 1988 through the present.

Jill Davidson, editor of *Horace*, welcomes your comments, issue theme and story ideas, and other feedback via email at jdavidson@essentialschools.org.

Lewis Cohen
Executive Director

Jill Davidson
Publications Director

"Creating a learning community that supports and encourages students' authentic engagement in the construction of mathematical knowledge depends primarily on the teacher's own efforts and instructional behaviors...If one hopes for students to develop the mathematical and social dispositions to act as a community of learners, then teachers must both support and model those ways of thinking and acting." -Azita Manouchehri and Dennis St. John, *Mathematics Teacher*, April 2006

"You can't divide something by a number if it's not given," stated Manuel during the D.T. In response, Jessica rebutted, "Yes you can - you can divide by a variable like x." Her counter-argument to Manuel's strikes at the heart of algebra. We can work with information even when numbers are missing. That simple statement informs me a great deal about Jessica's internalized understanding of the purpose of algebra.

Final Thoughts about Discourse Time

Experimenting in the classroom with an activity like Discourse Time involves a great deal of variability of several factors: the selection of a problem, the contributions of the students participating, the observational notes being taken, the scorer's accuracy, and the teacher's involvement. However, when D.T. goes well – and I can assure you from personal experience that that's not always the case – the learning that the community can take away is invaluable. Students gain:

- Deeper conceptual knowledge of mathematical ideas
- The ability to learn and apply new information
- Increased resourcefulness
- The experience of challenging each other's thinking
- The skill of determining what a question's really asking
- The experience of listening to someone else's opinion and synthesizing it with your own
- The ability to collaborate effectively
- The benefit of engaging in mathematical conversations

Such an experience allows students to explore what it's truly like to be a mathematician. Encapsulating the goal of developing creative, critical thinkers and problem solvers, Discourse Time is an essential component of an Essential school math classroom. As CES founder Ted Sizer writes in *Horace's Compromise*, "Understanding is more stimulated than learned. It grows from questioning one's self or from being questioned by others...Questioning is a far more difficult form of pedagogy for teachers than are coaching and telling, because it is the least predictable." Discourse Time is certainly not a predictable teaching practice. However, as responsive and progressive teachers whose efforts lie in preparing our students for the 21st century, we must embrace the value of such a practice. Dr. Sizer certainly has as he continues in *Horace's Compromise*. "Education's job today is less in purveying information than in helping people to use it – that is, to exercise their minds." Discourse Time is far less concerned with "purveying information" and much more interested in helping students to "use their minds well."

"All students need opportunities to talk about what they're learning: to test their ideas, reveal their assumptions, talk through the places where new knowledge clashes with ingrained belief" - Mike Rose, quoted in *Results Now.*

After graduating from the University of Hartford in 2003, David Singer moved to Denver to teach at Skyview High School, a large comprehensive high school. In 2005, he joined the design and implementation team for the creation of Skyview Academy High School, a small CES school in Thornton, Colorado and has taught there since the school's opening.

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Radical Math: Creating Balance in An Unjust World, Conference Report

by Jill Davidson

Founded in 2006 by Jonathan Osler, Math and Community Organizing teacher at El Puente Academy for Peace and Justice, a public CES high school in Brooklyn, New York, Radical Math is an organization for educators working to integrate issues of political, economic, and social justice into math education. The Radical Math website, www.radicalmath.org, is an evolving exploration of teaching and learning focused on both math skills and social justice issues.

In April 2007, Radical Math cosponsored "Creating Balance in an Unjust World" a conference on math education and social justice. Following school visits to several schools, including CES schools Fannie Lou Hamer High School, Institute for Collaborative Education, East Side Community High School, and El Puente Academy, the conference opened at El Puente. The opening session featured young people from around the United States talking about their experiences of understanding their world more completely through mathematical analysis. Students also participated in workshops the following day; Levon Kirkpatrick, conference participant and student at Vanguard High School, another New York City CES school, highlighted the necessity of mathematical literacy and the connection to real world contexts. "It allows you to know that math isn't stuff you do with just a piece of paper and a pencil. It has to do with life. I'm not asking my teacher 'Why do I have to know this?'"

With the urgent need for mathematical literacy and the current lack of equity in math education paramount in the consciousness of facilitators and participants, conference sessions included a variety of 28 workshops, two panels, and a keynote address delivered by civil rights activist Bob Moses, founder of The Algebra Project, a program that prepares underserved youth with high-level math skills. Moses is also co-author of *Radical Equations: Math Literacy and Civil Rights.* He spoke about the parallels between his prior work building demand for and securing voting rights in the 1960s and his more recent work building demand for high quality, equitable education through a focus on mathematics skills. "What's radical is doing math with the kids at the bottom of society and getting them to demand their educational rights," observed Moses. "When students begin to make a demand on themselves, they start to make demands on their schools and communities." Moses spoke about parallel efforts to build demand for a Constitutional amendment for quality education for all, mentioning the work of groups such as Quality Education as a Civil Right.

Related Resource

Horace Volume 20, Issue 22 (Winter 2004) featured "Working the Demand Side: An Interview with the Algebra Project's Robert Moses." www.essentialschools.org/cs/resources/view/ces_res/322

The conference closed with participants gathered in a variety of action groups, and a planning session for a subsequent 2008 conference. Initially, Osler and other organizers didn't plan on the conference becoming an annual event. "I had a naïve idea that we could spend time at the conference and then it would end. Instead, there has been an enormous amount of interest in this approach to math education," says Osler. "It would be great to make it happen in another city so other schools could be highlighted. That's what a good organizer does - we pass on those skills and keep it going." For those unable to attend, Radical Math will be issuing a DVD that offers conference highlights and handouts.

For more, please visit:

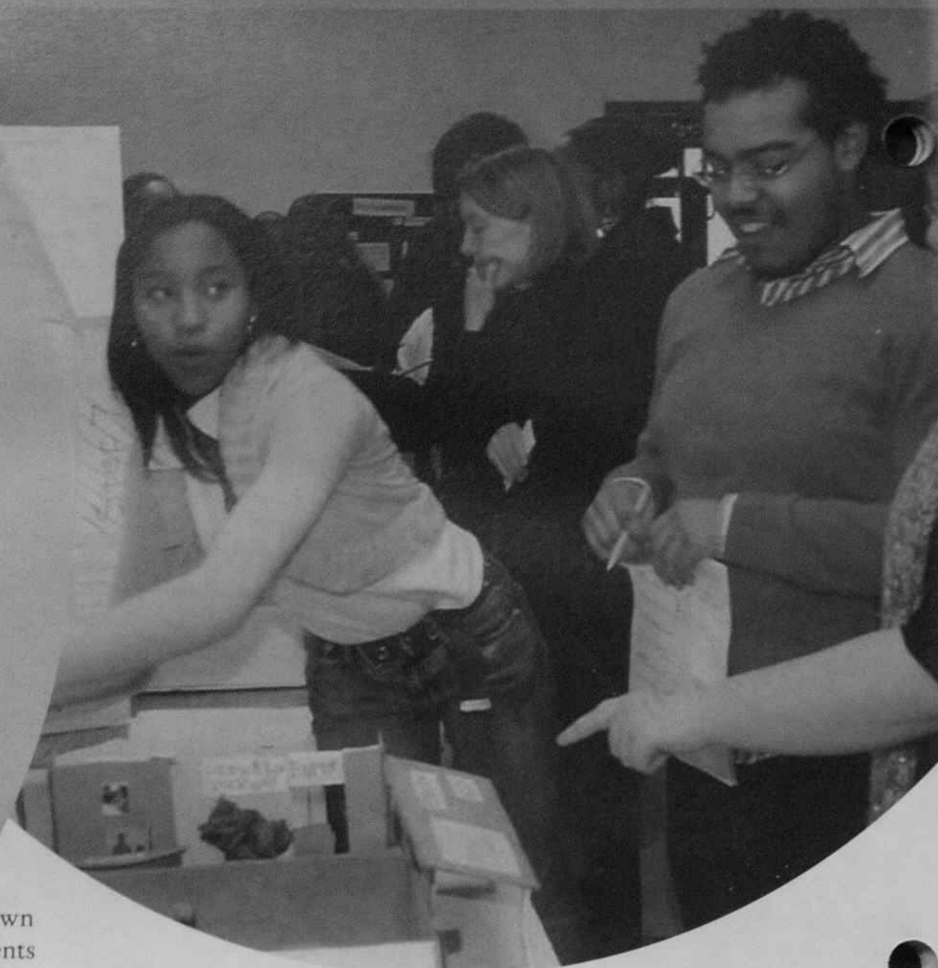
Radical Math
www.radicalmath.org

The Algebra Project
www.algebra.org

Quality Education as a Civil Right
www.qecr.org

The Case for Creativity in Math Education

by Mark Lonergan



"Mister, when am I ever going to use this?" When I started out as a new math teacher, these were the words I hated to hear because often, I wasn't so sure myself. In my own life as a math student, the sheer un-usability of the material was a big draw. As a teen, I could not figure out my life, but in this abstract world of math, I could resolve all my problems.

But once I started teaching, I realized that my own love of doing math was not enough. Some students got excited by the neat rationality of math. But most were uninspired. My internal motivators (love of math) and external motivators (desire to do well on the SAT tests) didn't translate to most of my students. So I began to rethink why it is important for all of us to learn math. What could I do to make all students see the beauty, fun and power in math?

Now, in response to questions about the importance of math, I explain that in our society, even though it may not be fair, math abilities equal smarts. People who can demonstrate skill with mathematics have more doors open to them than those who cannot.

And learning math is important because math was created by humans. When you trace the history and development of math, you really see the development of thinking. A math textbook, without always acknowledging it, contains wisdom from ancient cultures and the accumulated history of human thinking. We should all be part of that.

So after 11 years of teaching, the question I worry about now is "How do we do this well?" How do we engage students in math classes? How do we convince them that math is beautiful, fun and powerful? If we can engage students in the right way, then they'll be too busy to stop to ask, "When am I ever going to use this?"

For the math department at my school, Boston Arts Academy (BAA), the answer has been to infuse creativity into math classes. While still concerned with math standards and content, we focus on whether students have a chance to make choices and connect to their other passions. We want our students and teachers to feel like they are inventors as they create and solve problems. But what does it look like to be creative in math class, and why should we bother?

At BAA, being creative starts with the teachers. BAA is a pilot school within the Boston Public Schools, which allows us a lot of free rein to develop and implement our own ideas. My department and I have been able to create a curriculum that is still a work in progress. It's not yet beautiful, and it's far from realizing its full potential. But it is ours, and it grows and changes as we in the department continue to grow and change. As we've built this project-based curriculum, we have realized that after two or three years, a piece of the curriculum often needs to be improved or replaced because our thinking about what makes a good project continues to evolve. We teach new courses or have new perspectives on courses we've taught for years. The result: at every math level, we revise at least one project annually. Sometimes this means we throw out curriculum that works fine. Sometimes this means we try out new stuff that completely bombs. This can feel like building a sandcastle during high tide. We keep building new curriculum knowing that we'll never really be in a place where we feel "done." It can be frustrating, but the alternative – we decide to keep what we've got and make no changes – would mean that the department had stagnated and that we have stopped using our creative skills to develop new ideas. For us, being creative means taking risks, trying new things, and continuing to revisit and reinterpret what we're doing, even when we know we're doing it well.

One danger in being creative teachers is that we might veer off course, that we could confuse "new" with "good" and make wanton changes that are bad for students. This is one reason we recently hosted a half-day tuning protocol, which was a chance to get feedback on our evolving curriculum from professionals from other schools and members of the BAA community. The value of pausing to make our work public was twofold: First, we got feedback from outsiders. They told us that we're doing a lot of things well but that we should better articulate course themes and essential questions. To outsiders, some of our course descriptions felt like a random collection of projects instead of a cohesive learning experience. The second benefit of the tuning protocol was that the preparation required our department to work together, to find out what's going on in each other's classrooms and to think about what we're doing and where we would like to improve. The team had to articulate frustrations and turn them into focus questions. We had to document our work so that it could be comprehended by an outside audience. Teaching can often be such a private endeavor –we often don't know what's going on in the classroom next door. But by preparing for this tuning protocol, we were able to learn more about what we are doing and what we'd like to be doing in our classrooms.

We've realized that students also need to feel some sort of creative opportunity in order to feel engaged. This means teaching content within a project-based approach. We introduce traditional math skills, we practice those skills, and we take tests and quizzes on these skills. And we always make sure that students can apply those skills in meaningful ways through unit projects. In our best unit projects, there are four Cs that we consider:

CONTENT – We create our unit projects with rich math content at their core. We strive to find projects that not only require students to do good math, but in which doing the math is integral to completing the project.

CONNECTION – We make sure that there is an authentic, non-superficial connection to the real world and to other disciplines. At our school, this often means trying to connect our work to the visual and performing arts. But our best projects do more than just connect; they actually require students to use different disciplines and different multiple intelligences to complete the job.

CHOICES – Our best projects allow students to make choices and solve problems in different ways. There are chances for students to "muck around" with the math rather than just follow one linear pathway. This often means posing questions that do not have one "right" answer.

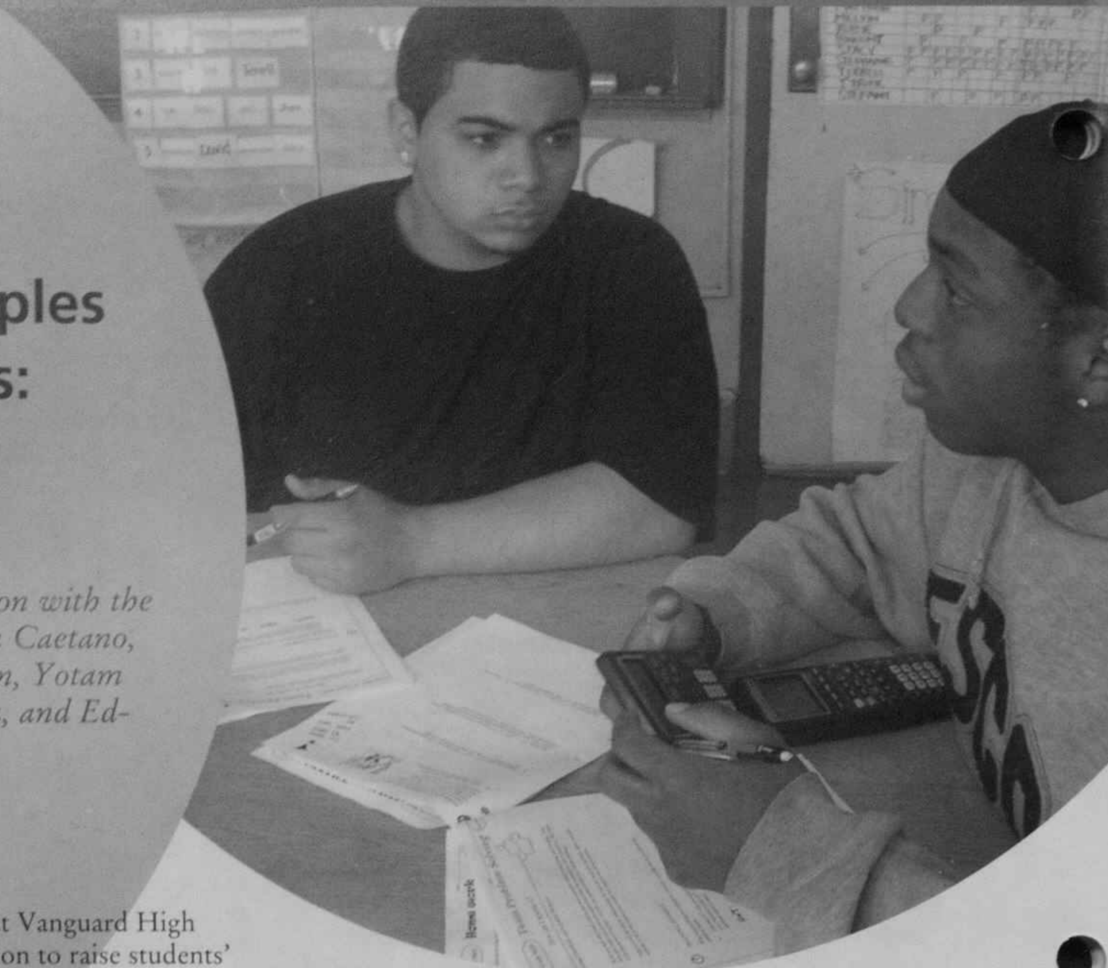
CO-AUTHORSHIP – We've found that students' engagement and achievement is highest when students can have a role in creating their problems as well as finding the solutions. Sometimes this means letting students change the setting or context of a problem, such as when Raphael transformed a boring problem about linear functions into a study about gas mileage that connected to his passion for motor bikes. Or this can mean giving students some constraints and having them come up with their own problems.

When combined properly, these four Cs ensure that math class can feel like a creative endeavor. We certainly don't have it all figured out. We continue to feel stretched for time and often feel in a tug-of-war between what we want to do and what we must do (preparing for state tests is the most significant "must-do"). We often run out of planning time and energy, so our best intentions and our philosophies don't always manifest themselves. And really, some topics benefit from a good old-fashioned lecture followed by traditional drilling.

But most days we feel like we're moving in the right direction. Although we've used different buzzwords throughout the years – arts-infused, project-based, multiple-intelligences, interdisciplinary – the consistent idea has been breaking the mold of traditional math classes and using our own creativity. When students and teachers can feel creatively connected to the curriculum, we circumvent the question, "When are we ever going to use this?" Instead, we focus on what we're going to create with these new skills. As teachers, we alsomodel the ability to create and support our students' ability to create on their own.

What I liked about math as a student is now what I don't like about it as a teacher. As a student, I loved math for its neat resolutions and final answers. As a math teacher, I've realized that I don't want neat conclusions anymore. I want to feel like the work I'm doing and the curriculum we're developing is moving in a certain direction, and remains a "work in progress."

Mark Lonergan is a math teacher and math curriculum specialist at the Boston Arts Academy. He joined BAA in 2000 after receiving his M. Ed. from Harvard Graduate School of Education. He grew up in Virginia and began his teaching career as an English teacher in Kumamoto, Japan.



Applying the Common Principles To Mathematics: Successes and Challenges

by Ankur Dalal, in collaboration with the Vanguard Math Team: Dorota Caetano, Rebecca Daczka, Lisa Gluckson, Yotam Hod, Jo Ho-Rolle, Kari Kokka, and Edgar Rodriguez

In the mathematics department at Vanguard High School, we have taken on a mission to raise students' interest, skill, understanding, and performance in mathematics Over the past several years, we have wrestled with the implications of the achievement gap, a pressing fact for us, as students enter our school well below grade level. Our own belief in education as an act of social justice, our adherence to the Coalition's Ten Common Principles, and our response to external requirements from the city and state has produced a mathematics program characterized by some notable successes, strategies and ideas for the future, and a number of lingering puzzles.

Vanguard's Math Program

The Vanguard High School mathematics program requires students to enroll in four years of heterogeneously grouped classes, culminating in a senior portfolio class. Students also have the option to take several math electives, including Mathematical Games, Architecture, and Calculus. Over the past several years, we have moved away from instructor-designed classes into a more traditional sequence of courses (algebra, geometry, advanced algebra, and more) to increase consistency in our department and free up time to focus on helping students. Students work collaboratively in groups, and the curriculum comes primarily from College Preparatory Mathematics, a program that emphasizes conceptual problems and spiraled content. Classes are small, generally 15 to 20 students each.

The primary challenge our department has faced is how to support a set of high academic standards for a population that enters our school well below grade level. Two years ago, an internal drive to raise the level of mathematical rigor and achievement at the school, coupled with city, state and federal pressure to raise student achievement and rates of credit accumulation and graduation, prompted our department to re-examine our curriculum and pedagogy, our portfolio and graduation requirements, our assessment system, our academic supports, and our opportunities for advanced work.

More than five years ago, Vanguard lost its waiver from the Math A Regents, the high-stakes, standard-

Vanguard High School is a small Essential school located in Manhattan's Upper East Side in the Julia Richman Education Complex. Vanguard serves approximately 400 students, and is 92.6 percent students of color (60 percent Hispanic and 30 percent African-American). Only 11% of students who enter Vanguard meet the state reading standards and only 28 percent meet the state mathematics standards. An unusually large percentage of the student body receives special education services.

ized graduation requirement in the state of New York. In response, our school responded by taking steps away from the Common Principles; we wrote our own curriculum that focused on covering the breadth of topics on the Math A (which includes a rather broad and unconnected range of material) and spent a month of each semester on test preparation. Many classes featured teachers in a traditional role of "deliverer of instruction services," focusing in particular on the tricks and sometimes obscure content requirements of the Math A exam.

While we were able to improve students' standardized test scores, we hadn't developed strategies to actually improve students' mathematical understanding. As veteran teacher Rebecca Daczka says, "It seemed like students weren't really learning – they were just memorizing for the test." Our faculty became frustrated that we often had to teach much of the same content at each grade level: each course had students re learning how to solve simple equations, and our Senior Portfolios focused on linear and quadratic equations, a topic more traditionally seen in ninth grade mathematics.

Many teachers in the department felt frustrated by the amount of time it took to write curriculum to cover Math A topics and offer projects and exhibitions. Daczka says, "I'd find myself spending two hours a night writing lessons, and I didn't have time to really find what my students understood."

Vanguard teachers were also at a loss for useful student assessment and achievement data. For many years, the department kept detailed grade reports that focused primarily on student work habits describing homework and class work completion, with commentary on exhibitions and projects. However, our assessment system did not provide some important information about our students. Many students failed our classes, and their grade reports often reflected poor work habits, especially regarding homework.

Related Resource

In 2005, Vanguard High School and 27 additional schools belonging to the New York Performance Standards Consortium won a court ruling allowing a five-year waiver from the New York State Regents standardized exams. An overview of this legal victory in defense of performance based assessment, "Making the Pendulum Swing: Challenging Bad Education Policy in New York State," by Ann Cook and Phyllis Tashlik, was published in *Horace* Volume 21, Issue 4, Fall 2005, www.essentialschools.org/cs/resources/view/ces_res/380

However, during conversations and class time, we'd often find a number of these students really understood the material. On the other hand, students could have excellent grades given good work habits, diligence about tutoring, and seeking help from classmates, and yet have very weak mathematical skills and conceptual understanding. For instance, this past year our department watched a video of a student during his senior portfolio struggling with basic concepts and definitions that should have been addressed in the ninth grade. His impeccable homework habits and conscientiousness about seeking help, coupled with the high grades this produced, obscured the need for remediation until this portfolio presentation.

A Chance to Reflect

Over the past two years, a number of external accountability changes and inte rnal reflections have precipitated a chance to reflect on some of the challenges we have been facing. We received a new waiver from the Math A exam, but at the same time, we received demands from the city School Quality Review program to develop a system of interim assessments for the students called a Design-Your-Own Interim Assessment (DYO). While the waiver exempted students from the high-stakes aspect of the Math A exam, we still needed to offer the exam to meet federal No Child Left Behind requirements (the Federal government does not recognize a waiver, which was offered by the State of New York).

Our department viewed this as a chance to think about our graduation requirements and portfolio assessment system. We expressed the need to develop new performance assessments suitable for every grade level. We also decided to raise the level of rigor in the higher grades, while acknowledging a need to improve our curriculum to spiral content and increase conceptual understanding. We focused on improving our assessment and remediation strategies to discover student weaknesses and attempt to fix them systematically.

These last two years have been a time for reflection, and we have developed a number of strategies to address our challenges, described below in terms of some of the CES Common Principles.

"Learning to Use One's Mind Well"

The first step in raising the level of our graduation requirements was to define what we meant by rigorous mathematics, or, in the language of the Common Principles, what did it mean for our students to use their minds well? Our department chose to return to Vanguard's Habits of Mind, which focused on the need for students to learn the process of thinking well. This process requires students to reason with

Continued on next page

evidence, offer conjectures, seek significance, make connections, consider viewpoints and think about their learning.

Of course, this process is in service of learning particular content. Our department adapted a curriculum called College Preparatory Mathematics that focuses on grade-level standards, fewer, more conceptual questions, skill practice spiraled throughout the years and cooperative learning. We have begun to rewrite the content expectations of this textbook series in terms of our habits of mind, resulting in a list of understanding goals. For instance, one of our understanding goals comes from a tenth grade unit on Quadratic Equations, and it requires "students to make connections between verbal, tabular, graphical and analytical representations of quadratic situations and reason with evidence about how to go from one representation to another."

"Less is More, Depth over Coverage"

We selected a curriculum that spirals content, revisiting major topics many times over four years with increasing depth, and standardized understanding goals in terms of our habits of mind. As we started this process, we noticed that we've developed a set of expectations that emphasizes deep thinking. For instance, an eleventh grade understanding goal concerning Quadratic Equations requires "students make connections between verbal, tabular, graphical and analytical representations, including standard, factored and vertex forms, of quadratic situations"; students often revisit the same understanding goals in different years, simply with greater depth and newer material.

Furthermore, an emphasis on writing standards in terms of a few habits of mind pushes students to employ fewer, more generative ways of thinking rather than memorizing a plethora of mathematical facts, rules and tricks. An understanding goal from Calculus also requires students to make "connections between verbal, tabular, graphical, and analytical representations," but this in service of understanding derivatives.

"Goals Apply to All Students" and "Personalization"

In writing these understanding goals, we have begun to write standards that apply to all students. In previous years, our department often had difficulty expecting the same understanding from all students. While all students had exposure to the same material, we tended to alter our goals for our performance assessments for our weaker students. While we made these adjustments to ensure all students had a chance to pass our courses, we failed to offer those "students not yet at appropriate levels of competence ... intensive support and resources to assist them quickly to meet [our] standards," primarily because we did not have a system to identify the source and nature of students' individual difficulties.

Currently, our department is in consultation with Vanguard's Instruction Support Services team to identify those core understanding goals that will mark the heart of our mathematics experience, those goals that will apply to all students. Additionally, we are working to develop scaffolds and adaptations. Finally, we are designing our assessment system to maximize personalization of learning and teaching; students attempt to demonstrate mastery of understanding goals whenever they are ready, not just on test days or at the end of a unit. Students certainly appreciate this level of personalization; sophomore Evelyn Santiago says, "[Understanding checks] changed my way of thinking about math a lot because now I think you can keep trying until you understand something."

"Demonstration of Mastery"

This Common Principle suggests "multiple forms of evidence, ranging from ongoing observation of the learner to completion of specific projects, should be used to better understand the learner's strengths and needs, and to plan for further assistance. Students should have opportunities to exhibit their expertise before family and community." As such, we have developed a number of different assessment vehicles through which students can

The Vanguard Habits of Mind

Working with students and staff with the intellectual focus on "using their minds well," our curriculum is developed stating the concepts, skills, knowledge and goals of year-long or semester-long thematic courses. We use the "Habits of Mind" to plan and implement curriculum. These are:

How well does the student use evidence to support his/her opinions or conclusions?

Does the student see other points of view?

Can the students see connections between different topics, areas and courses?

Is the student able to see the relevance of our studies?

Does the student see various factors in an argument and conjecture what would happen if they were changed?

In this process, students are expected to show and deepen their understanding of the five main "Habits of Work:" punctuality, organization, cooperation, revision, and focus.

demonstrate their understanding.

Beyond written tests of the understanding goals, which we offer several times a semester as a part of our DYO interim assessment system, we have developed individual oral defenses, roundtable discussions and individual assessment interviews. In the oral defense, which we have expanded to the tenth and twelfth grades, students individually present and defend their knowledge through an explanation of an authentic mathematical task, followed by intense question and answer time. In a roundtable discussion, students work in small groups to tackle a new and novel situation; at the end of the roundtable, there can be an opportunity for evaluators to interview students to reflect on the process and develop a fuller sense of the student's understanding. Finally, assessment interviews provide an alternative to written tests for those with test anxiety or language difficulty; students are evaluated during a live interview instead of through a written test.

While students find the demands of mastery more difficult, many report appreciation for this way of learning and assessment. Student Jeffrey Rodriguez says, "I like to know that I am getting graded on how well I understand. It's harder, but we're proving that we know how to understand things."

"Student-as-Worker, Teacher-as-Coach"

To meet these standards, we've begun to share pedagogical strategies "to provoke students to learn how to learn and thus to teach themselves." Some of us have adopted a cooperative learning strategy developed by researchers at Stanford University called Complex Instruction, in which students are assigned roles and teachers work to provide group-worthy tasks and mitigate perceived status differences in students' abilities by valuing student questioning, skepticism and demands for evidence from fellow classmates. We have also started to implement study team strategies, such as participation quizzes, in which students are given a daily grade based on the quality of their conversations as recorded by the teacher on an overhead. We also utilize team tests, in which all students at a study team receive the same grade for a test, encouraging them to take ownership over their own learning.

"Democracy and Equity"

The changes and strategies listed above are part of a larger vision of democracy and equity: a belief in education and social justice that strives to offer an underserved, urban population of color educational opportunities that are equitable to those granted to wealthier suburban students. Our department has changed our exit requirements to focus on grade-level content, and we have developed a Calculus class for students motivated to extend their study of mathematics. Students who once failed middle school mathematics are now successfully completing a course in calculus.

Lingering Puzzles

Our department has reached a few conclusions about our work. While we generally enjoy the increased rigor and consistency provided by our outside curriculum, we sometimes need to develop larger projects appropriate for our performance assessments. Each grade team spends part of every week collaborating on adaptations of the curriculum to meet our needs. In our department meetings, we have conversations about how to adapt the curriculum for our special needs children. We've also spent department meeting time working on analyzing our student performance information and assessment data, and we're trying to rearrange our staff to create additional math support classes based on what individual students need to know.

Though we are excited by the changes we've implemented over the past few years, a number of lingering puzzles remain. For instance, how can we use the information we collect on each student's understanding to better inform our remediation system? Currently, when students fail classes, they are placed in a generic night school or summer school program; how could we provide a more individualized experience so students learn what they don't understand?

We also wonder how to write standards that truly emphasize deep, meaningful thinking and not just superficial knowledge. In working with our understanding goals, we've found some to be too procedural or obscure. How can we be sure our standards get at the heart of our subject matter?

We also wonder how these strategies can translate into improving students' standardized test scores. Many of our students have difficulty scoring well on external metrics (although they have improved considerably on the Math A exam). What can we do as a school to promote success on these tests?

Finally, we want to know how we can use the information we collect about student achievement to inform our own practice. How can we reshape units based on our students' demonstrated understanding of topics? How can we work as a team to ensure students can demonstrate mastery of our goals?

Ankur Dalal is finishing his second year at Vanguard High School. Previously, he taught at San Lorenzo High School in San Lorenzo, CA. He graduated from Stanford University and studied Computer Science, History and Mathematics Education.

One Instructor's Quest for a Collaborative Professional Culture

by Jimmy Frickey

With a B.S. in math but no prior math education training, my first job as a math teacher was at an alternative charter school with a holistic mission. I was charged with teaching math to five groups of 17 7th, 8th, and 9th graders, embracing problem solving, communication, and collaboration. Few of these students had been taught math in this way previously, and I had not taught math this way (or any other way, for that matter). While I still believe such an approach is possible and worthwhile, I was not prepared to pull it off. I struggled tremendously and no doubt left numerous opponents to math reform in my wake. Fortunately, I attributed my ineffectiveness to my lack of experience and skill as a facilitator and curriculum writer, not to a flaw in the vision. Though I have no way of knowing, I have since wondered what percentage of new educators in similar situations would draw a different conclusion, something like, "Math is different from other subjects. It can't be learned collaboratively. You just have to memorize." This experience motivated me to understand why and how math teachers can become effective in settings that stress understanding over memorization.

Why Did I Struggle?

I believe I struggled in the effort to engage students in the work of actually doing math, as opposed to practicing other peoples' math, not only because I was a new instructor, but also because I learned math quite differently from the way I was aspiring to teach it. My earliest school memories are of racing through addition and multiplication charts to win ice cream from my teacher. I remember doing pages of practice problems and receiving high percentage scores for getting most problems right. I do not remember ever applying math to real world problems with more than one correct answer, and I never remember working with other people on a problem. Interestingly enough, the only times I remember solving problems at the board in school were when I was racing to be the first finished with a correct answer in class competitions. Incidentally, I loved those days – no doubt because I usually won!

If memories like mine approximate the math experiences of others among the current group of math educators in the CES network, is it any wonder we struggle to create math environments where students solve meaningful problems collaboratively with others? If I, or any math instructor, haven't actually experienced learning important math concepts by solving and discussing problems in a collaborative environment, how do I even know it is possible? And if I don't believe it's possible for all students to learn math without an expert telling them what to do and encouraging (often coercing) them to practice and memorize, how much creative energy and perseverance can I bring to the challenges of engaging all students with a meaningful, creative mathematics education? With the benefit of a few years of experience, it now seems obvious that I struggled, and I believe many of us struggle, because I had almost no actual experience learning math concepts through problem solving in collaborative settings.

In my classes of 17 students ages 12 to 15, I was encouraged to select problems for students that were "open-ended" and offered a variety of solution strat-

egies that allowed, at least in theory, students with differing math backgrounds to engage productively with the problem. As students with vast differences in prior experience and confidence asked questions about the problems, as well as the new teaching style, I clumsily directed them to "rely on each other." Within the first 20 minutes of my first lesson ever, the students figured out that I did not intend to answer all of their questions directly. Over the next few weeks, students, parents, and possibly colleagues questioned my understanding of the role of "teacher," sometimes with a real intent to understand where I was coming from, other times rhetorically, with disdain and judgment. "Teachers, especially math teachers, answer students' questions," they reasoned. "How am I supposed to know what to do, if you don't tell me?" "What should my student do when she is ready to move on and others aren't?" "How can someone struggling with fractions really work productively with my 'Algebra' student?"

These questions, I believe, represent something more than most first-year teachers endure. They reveal the responsibility reform-minded educators share in helping others come to understand why reform is warranted and worthwhile. This responsibility is not appropriately borne by any first-year teacher. Despite my best intentions and excitement, I failed my first year teaching math for two primary reasons. I failed because I was unskilled at facilitating a collaborative problem solving culture and had few prior teaching examples in my learning history to imitate. And I failed because I was unprepared to manage effectively the demands of first year teaching, curriculum development, and the necessary public relations work of a new charter school.

Before I discuss the nature of my failures, let me say that I resist the temptation to blame my employers. First, creating a new school is a tremendous amount of work and there was much they did to support me, including sending me to the National Science Foundation (NSF) funded Interactive Mathematics Program (IMP) training. Second, and this is my main point, I don't believe they understood the nature of the challenge. Because of that, to my knowledge, the right kind of professional development networks did not exist that could have adequately prepared me for the challenge I was facing. As good as the IMP training was (and still is), it was not designed with small, holistic-minded community based schools in mind, and I was therefore left to synthesize what was essential in the training with what was essential to teaching within the philosophy of my school. This is the task that I now believe is too large for any first-year teacher.

My failures manifested themselves in two ways. First, few students made significant gains in mathematics that year. Second, most people attributed the lack of learning to the change in instructional philosophy. That is, my first year of teaching helped reinforce what many students, parents, and educators believed in the first place. "Math is different from other subjects. Math is memorization and practice. Math is not creative." And, worst of all, "Some people are good at math, and some people aren't."

But I refuse, and continue to refuse to believe the conclusions so obvious to many students, parents, and colleagues, because I have had two experiences learning important mathematics through problem solving in a collaborative culture. The first experience was in college in an Abstract Algebra course. Our instructor gave us problem sets designed for us to discover, explain and prove the major theorems of Abstract Algebra. He answered our questions with questions of his own and forced us to rely on each other for validation. We even took two group exams! The second experience was the IMP training mentioned above, a two-week course provided as professional development. Sixty secondary math instructors, some converted art teachers with outright math anxiety, others, like me, with degrees in mathematics, discovered that much of what we thought we knew about probability and expected value was in fact quite superficial and unable to be applied accurately to non-routine problems. Collectively, we expanded our notions of what it meant to understand a mathematical concept.

These experiences, and others since, form the foundation of my beliefs that 1) math can be learned through solving problems collaboratively with others, and 2) the math I thought I learned previously lacked the depth and flexibility of true knowledge.

Forming a Network

After my first teaching experience, I decided to opt out of the "sink or swim" model of professional development offered by so many of the new small schools sprouting up across the nation. I pursued an internship at Eagle Rock School with the hope of learning to teach math alternatively from an experienced educator. While Eagle Rock teacher Jason Cushner fit (and continues to fit) that description, as I was applying, he was making plans to leave. Jason worked at Eagle Rock School for seven years as the Instructional Specialist in mathematics prior to my application to Eagle Rock and had even received the Presidential Award in Mathematics Education in 2002. I was offered the position and worked with the previous year's intern while the school looked to fill Jason's position.

Fortunately for me, the position was still vacant seven months into my internship and the job was offered

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Essential Mathematics Education

Notes on This Issue

One of the most wonderful aspects of editing *Horace* is having the opportunity to collaborate with CES educators willing to write about their teaching, their students' learning, and their professional collaborations and growth. This issue offers the work of a group of math teachers passionate about their challenges as well as their achievements. They are inspiringly focused on equitable attainment of meaningful, powerful math learning for their students. And they care about building the strength of math education throughout the CES network.

Several times throughout the months of working with the dozen teachers who contributed to this issue, I wanted to jump the fence and join them. Nothing sounded as compelling, intriguing, and fun as teaching math (and I am a humanities teacher by training!). You'll be swept up by their passion, enthusiasm, critical insight, and desire to improve and refine how we teach math in Essential schools. Many thanks for looking in and sharing out to all who contributed to these pages.

Jill Davidson
Editor, *Horace*

The CES Graduate Vision

- CES graduates will continuously seek to understand the world around them in an effort to meet the needs of self and community and have the moral courage and integrity to live the life that fulfills their passions.

- CES graduates will be knowledgeable, curious and passionate problem solvers who continually learn about, participate in, and enjoy life in the community and world around them.

The CES Math Classroom Vision

CES Math Classrooms:

- Establish and maintain a positive culture of learning where students are working together and are developing confidence in their math abilities and understanding.

- Set clear learning objectives/goals for each class or lesson so that students and teachers know what they are learning and why.

- Differentiate instruction and assessment to meet the needs of individual students and provide multiple opportunities to demonstrate understanding.

- Keep it real, relevant, and authentic

- Personalize instruction

- Practice the 3 Ds: demonstrate, debate, and defend

- Emphasize mathematical culture and discourse

- Are relevant – interesting, applicable and integrated

- Challenge all students

- Actively engage students by developing a community of learners

- Require students to demonstrate/justify understanding

to me. While this was a tremendous opportunity, the hope of learning to teach math alternatively from an experienced educator never played out for me. Even with Eagle Rock's belief in the "Teacher as Generalist" principle, there were few, if any, people offering experience and knowledge about how to facilitate a collaborative problem solving class in a heterogeneous environment, how to develop and utilize inter-disciplinary courses, and how to re-engage students with mathematics. On the contrary, in my perhaps overly cynical moments, I sometimes got the sense that people were just glad someone at Eagle Rock was willing to do math.

So I began to look outward. I invited instructors I admired from professional development experiences outside of school to observe my courses, collaborate with me to address challenges unique to Eagle Rock, and help identify and recreate our successes. At the debriefing of one such experience, the idea of creating the community of math instructors I was craving was born. While musing about how most math conferences talk about things meant for dramatically different schools settings and how most internal school professional development focused on anything but implementing math curriculum consistent with our school vision, my guest suggested I invite people to Eagle Rock for a conference. Eagle Rock is in beautiful Estes Park, Colorado, he reasoned, and people would love to come here to "work on math."

Math Innovators' Forum

As a result, Jason Cushner and I created the Math Innovators' Forum. The Forum's purpose is threefold: 1) improve the collective practice of math education in alternative schools, 2) support new instructors in the Herculean task of being a new math instructor in an alternative school, and 3) to advocate for our agenda of improving math instruction in our schools. For two years, Jason and I have worked to connect math educators with one another in small alternative schools in the belief that such work improves our practice and is desperately needed. We have hosted four retreats connecting with about 30 instructors. Of particular interest to us are the several brand new instructors who attended and seemed extremely thankful to have someone, anyone, with whom to work on math.

At these gatherings, we identify challenges common to our situations and work to share effective ideas and resources. In the first few Forums, we worked to develop curriculum and projects together. More recently, we have examined existing lessons and projects with examples of student work. Participants who bring work to be reviewed agree to make any improvements to the assignment based on the discussion, and we disseminate the work to the larger group. The goal is to build a significant collection of lessons, projects, assessments, rubrics, and other material that can be used as a resource, particularly for new instructors striving to make math more meaningful for their students.

To learn more about and participate in the Colorado-based Math Innovators' Forum, contact Jimmy Frickey at jfrickey@eaglerockschool.org or Jason Cushner at jcushner@hotmail.com. The Forum meets twice a year, in the fall and the spring, and is exploring ways to incorporate more school visits between members and other smaller gatherings to increase dialogue and communication.

CES Math Super Team
I discussed these ongoing efforts with Skyview Academy math teacher David Singer, a colleague from the CES network. David suggested we pursue the same idea nationally within CES. From that conversation came "Developing a CES Math Super Team: Collaboratively Solving the Math Dilemma," a preconference session on mathematics in CES schools at the 2006 Fall Forum in Chicago, an email listserv on which math educators can post questions and discuss math-related challenges and share successes with each other, and a space on the CES ChangeLab site where instructors can post lessons, activities, and video clips of successful curriculum and activities.

Spending the Fall Forum pre-conference day with more than 25 other instructors from similar school environments interested in improving math education within the CES network was a tremendous gift. I left the experience with a renewed commitment to keep math relevant and meaningful for all my students. We began the day together defining a collective vision of a CES graduate, and we then articulated a vision for CES math classrooms in support of our vision for CES graduates.

Insights
The experiences of isolation relieved by collaboration have led me to a few insights about my practice as a math educator and math education in small, personalized, and student-focused schools:

Effective instruction requires collaboration with other instructors.

There are people working effectively to implement alternative math programs, but they are exceptions to the rule and they are not currently in positions to share their wisdom with people who could benefit.

New instructors need experiences that will ground their belief in the idea that math can be learned through collaborative problem solving, or else they will teach the way they were taught. If that support isn't present in your school, then it is our responsibility to make it available.

Curriculum writing is hard! We need to collaborate and share with one another so instructors can spend more of their time focusing on questioning and assessment strategies in support of collaborative classroom cultures.

In our brief time in Chicago, it was clear there are many areas of common interest and concern: developing more inter-disciplinary classes and projects, developing additional classes and projects connecting math to issues of social justice, wrestling with the challenges of heterogeneous groupings, understanding and implementing a "less is more" philosophy in the current "high-stakes" climate of accountability, and developing collaborative, inquiry-based classrooms, to name a few.

To create connections with other math teachers within CES, join the CES Math Superteam email listserv. Here's how:

Visit www.essentialschool.org

Log in. If you have not ever created a CES Interactive account, you will be prompted to do so.

Click on the My Homebase tab

Join the Math Superteam discussion by entering Math Superteam into the Find a Discussion search box

Start connecting!

The Math Superteam can also be accessed at www.essentialschools.org/cs/homebase/forum/cs_disc/1565

I'll close with some questions that suggest some thoughts about how we can continue and expand this work. What if we could gather, develop, and share five ways linear relationships have been successfully taught in interdisciplinary courses? What if we knew who within CES really understands how to teach spatial reasoning and made her knowledge readily available? What if we collectively advocated for a more complete, humane, and useful vision of mathematics education? What if CES offered training in math education meant for smaller community-based schools? What if CES collaborated to bring math educators together in the summer to develop curricula and offer intervention courses to students who need them at little or no cost in support of high expectations for all students and with the goal of sharing that curriculum?

With every challenge we face, there is an opportunity. The fact that so many have such similar opportunities suggests we are dealing with something larger than our own schools and classrooms. Individual successes in the network can create future allies, and more successes can add to the momentum.

Jimmy Frickey is Math Instructional Specialist and houseparent at Eagle Rock School and Professional Development Center. He has a B.S. in mathematics from Miami University and has been teaching math for six years.

Collateral Damage: How High-Stakes Testing Corrupts America's Schools *by Sharon L. Nichols and David C. Berliner* (Harvard Education Press, 250 pages, $24.95), reviewed by Jill Davidson

Collateral Damage will recommit you to the Common Principles like a drowning person recommits to respiration. Nichols and Berliner provide a satisfyingly reasoned critique of high-stakes standardized tests, a tight, research-based, evidence-rich argument that creates urgency for the widely held conviction that our education system must shift toward more human-scale ways of measuring what students know and can do.

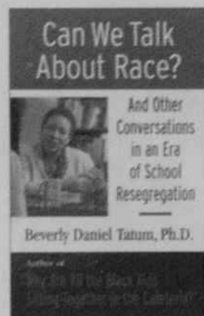
Collateral Damage will infuriate you with its train-wreck accounts of cheating, student neglect, data misrepresentation, distortion, corruption, and corrosive damage affecting not only individual students, teachers, and schools, but the entire enterprise of American public education. We all live with the compromises, wasted time, anxiety, anger, and losses that accompany the ubiquitous high-stakes standardized tests. *Collateral Damage* pulls together hundreds of such instances, revealing the horrifying parameters.

The authors reinforce their assertions with Campbell's law, the social scientist's version of the Heisenberg uncertainty principle. Campbell's law stipulates that the more a social indicator is used for decision-making, the more likely it will be corrupted and the social processes it is meant to measure will be distorted. Nichols and Berliner argue that the current over-reliance on testing creates a frighteningly ideal environment in which Campell's law can operate.

We know the insidious effects. Educators view certain kinds of students as liabilities. To improve scores, the system pressures educators to ignore these young people and push them out, creating a cruel world in which the students who most need help are most likely to be denied it. We have drastically narrowed the curriculum while undermining teachers.

Collateral Damage serves as a validation of demonstration of mastery, emphasizing the integrity and validity of exhibitions and other incorruptible authentic work completed over time. Nichols and Berliner endorse such assessment work as one of the likely alternatives to the current situation; sharp eyes will note that all examples in this section are all CES schools (and CES is mentioned by name). Even in this nearly airless climate, CES educators can inhale, at least a bit. We know that we must collectively demand a different system, and we who are in CES schools are, in fact, at the forefront of creating that better system.

Can We Talk about Race? And Other Conversations in an Era of School Resegregation *by Beverly Daniel Tatum* (Beacon Press, 168 pages, $22.95), reviewed by Jill Davidson



Beverly Daniel Tatum argues that we must talk about race in order to combat insidious school resegregation and make good on the promise of our nation's diversity. Policies reinforcing residential segregation patterns and federal judicial decisions that release school districts from previous court-ordered desegregation plans are producing a generation of schoolchildren with less reliable daily contact with people of other races than their parents had. "Meaningful opportunities for cross-racial contact are diminishing, especially in schools." Tatum forcefully describes resegregation's disastrous effects: concentrations of poverty or wealth, lack of meaningful contact among members of different racial groups, and the injury individuals and our nation suffer when race-sorted cohorts of children miss out on the education to which they are due.

Even as she establishes this bleak big picture, Tatum delivers her key message: we are the stewards of our multiracial heritage, and we must act. We can talk about race and we must consciously shift our priorities to do so. As she describes resegregation and the negative effects of unexamined racial assumptions on student performance, Tatum covers familiar ground. But the central question, "What can we do about this?" makes this disturbing material compelling. You can develop "the ABCs of inclusive learning environments: affirming identity, building community, cultivating leadership." We can do the hard work of uncovering unconscious beliefs about White, Black, and other children.

Tatum reinforces the effects of knowing students well, holding students to high standards, and supporting them in personalized ways. She also examines the promise and challenges of interracial friendships. "Human connection requires familiarity and contact," she reminds us. "Connection depends on frankness, and a willingness to talk openly about issues of race." Contact among diverse groups of students is necessary, but not enough. Knowing how to talk about race is fundamental for meaningful connection.

Can We Talk about Race? reinforces our ability and commitment to talk about race, emphasizing that the kind of race-conscious, personalized, achievement-driven schools that we're creating are good for kids – and for ourselves. As we talk, we strengthen the equity gains that are the legacy of the ongoing demand for civil rights and freedom for all.

Where to Go for More

The Math Forum
Like CES, the Math Forum is collaborative, built by and for math teachers. The Math Forum is a long-standing, immense library of tools, curricula, teaching techniques, assistance, analysis, and opportunities for interaction to improve math teaching and learning. Run by Drexel University's School of Education, the Math Forum serves teachers and learners with some for-fee services and many free resources. The Math Forum is terrific for personalization, providing materials suited to individual learners as well as the big picture of group curricula along with lesson exchanges and active online discussions on a wide range of math education topics.

The Math Forum
3210 Cherry Street
Philadelphia PA 19104
phone: 215.895.1080
www.mathforum.org

Rethinking Mathematics
Rethinking Mathematics: Teaching Social Justice by the Numbers, edited by Eric Gutstein and Bob Peterson, published by Rethinking Schools in 2005, serves as a clarion call to educators seeking to integrate social justice with meaningful, academically challenging mathematics learning. Though the accompanying website isn't a substitute for the book, it provides a generous selection of excerpts and worksheets.

Rethinking Schools
1001 E. Keefe Avenue
Milwaukee, WI 53212
phone: 414.964.9646
www.rethinkingschools.org/publication/math

Mathematical Association of America
Though primarily devoted to undergraduate-level math, the Mathematical Association of America (MAA) serves as a guide to the world of college math into which our students are entering. The MAA spreads into the world of K-12 education with its Common Group reports describing the MAA's efforts to encourage and facilitate discourse between mathematicians and educators to improve K-12 mathematics teaching and learning. SIGMAA QL, its quantitative reasoning interest group, is an exciting resource for deepening mathematics literacy and understanding.

The Mathematical Association of America
1529 Eighteenth Street, NW
Washington, DC 20036-1385
phone: 202.387.5200
www.maa.org

The K-12 Mathematics Curriculum Center
The K–12 Mathematics Curriculum Center aims to help teachers and administrators make thoughtful, informed decisions about mathematics curricula. If you're seeking or considering adopting a new curriculum, the Center's resources will be crucial to your decision. The Center offers a range of ways to examine the potential of various curricula – video overviews, research reports, teacher narratives, and summaries.

K–12 Mathematics Curriculum Center
Education Development Center, Inc.
55 Chapel Street
Newton, MA 02458-1060
phone: 800.332.2429
www.edc.org/mcc

Center for Proficiency in Teaching Mathematics
Funded by the National Science Foundation, the Center for Proficiency in Teaching Mathematics (CPTM) focuses on two questions: What mathematical knowledge and skill is needed for the effective teaching of mathematics? How can teachers develop and learn to use this knowledge and skill in their professional practice? CPTM sponsors events, study groups, research projects, and in-depth study and training opportunities aimed at improving K-12 and collegiate math education.

Center for Proficiency in Teaching Mathematics
1600H School of Education, 610 East University
The University of Michigan
Ann Arbor, MI 48109-1259
phone: 734.615.9048
email: cptm@umich.edu
www.cptm.us

and

105 Aderhold Hall, College of Education
The University of Georgia
Athens, GA 30602-7124
phone: 706.542.4194

Math for America
Math for America (MfA) runs the Newton Fellowship and Master Teacher Programs, generous high school teacher training programs for aspiring and mid-career educators. MfA's website is a useful clearinghouse of resources related to mathematics education, student achievement, and teacher quality, with a strong practical section on teaching math in student-centered ways.

Math for America
50 Broadway, 23rd Floor
New York, NY 10004
phone: 212.206.0053
email: information@mathforamerica.org
www.mathforamerica.org

Go To The Source: More about the Schools and Other Organizations Featured in this Issue

Schools

Amy Biehl High School
123 4th Street SW
Albuqueque, New Mexico 87102
telephone: 505.299.9409
www.abhs.k12.nm.us

Boston Arts Academy
174 Ipswich Street
Boston, MA 02215
telephone: 617.635.6470
www.boston-arts-academy.org

Eagle Rock School and Professional Development Center
2750 Notaiah Road
P.O. Box 1770
Estes Park, CO 80517-1770
telephone: 970.586.0600
www.eaglerockschool.org

El Puente Academy for Peace and Justice
211 S. 4th Street
Brooklyn, NY 11211
phone: 718.387.0404
www.elpuente.us/homepage.htm

New Mission High School
67 Alleghany Street
Roxbury, Massachusetts 02120
telephone: 617.635.6437

North Central Charter Essential School
1 Oak Hill Road
Fitchburg, Massachusetts 01420
telephone: 978.345.2701
email: info@ncces.org
www.ncces.org

Parker Charter Essential School
49 Antietam Street
Devens, Massachusetts 01434
telephone: 978.772.3293
www.parker.org

Skyview Academy High School
9000 York Street
Thornton, Colorado 80229
telephone: 303.853.1900
www.skyviewacademy.us

Vanguard High School
317 East 67th Street
New York, New York 10021
telephone: 212.517.5175
http://vanguard.r9tech.org

Other Organizations

College Preparatory Mathematics
1233 Noonan Drive
Sacramento, CA 95822
telephone: 888.808.4276
www.cpm.org

Complex Instruction
Stanford University School of Education
485 Lasuen Mall
Stanford, CA 94305-3096
telephone: 650.723.2109
www.stanford.edu/group/pci

Interactive Mathematics Program
P.O. Box 2891
Sausalito, California 94966
telephone: 1.888.MATH.IMP
www.mathimp.org

New York Performance Standards Consortium
317 East 67th Street
New York, New York 10021
telephone: 212.570.5394
email: info@performanceassessment.org
www.performanceassessment.org

CES Announcements

CES Summer Institute 2007

The Essentials of Small Schools: Principles and Practices for Equity and Achievement
Miami Beach, Florida
July 9-13

The Coalition of Essential Schools, Southeast CES Center, Nova Southeastern University, Fischler School of Education and Human Services, and the Florida School Choice Resource Center invite you to join us for our 2007 Summer Institute at the Miami Beach Resort and Spa in Miami Beach, Florida.

The CES Small Schools Summer Institute features workshops and facilitated conversations with some of the most effective small school educators in the country. Participation is open to individuals and school teams who are starting new small schools, working in large schools converting to small autonomous schools or small learning communities, or who are looking for a community of experienced practitioners from whom to learn and share best practices. Planning time for teams is built into the week, as are meals and special events around Miami.

Ongoing sessions and opportunities for youth involvement are key features of Summer Institute: students are strongly encouraged to attend with adults from their schools

On Wednesday, July 11, 3:00-6:00, there will be a special Symposium, Redesigning Large High Schools for Equity. The Symposium will be an interactive panel that will include national and local experts on high school redesign. This special event will examine the strategies of converting a large comprehensive high school into small autonomous schools or small learning communities. The speakers will focus on creating both school and district level change in order to create and sustain equitable, personalized, and intellectually vibrant schools.

The Symposium will be followed by a reception for speakers and attendees. The Symposium and reception are included for all Summer Institute attendees and are available at a low cost for other interested educators and community members.

Register now at www.essentialschools.org! Early bird discounts are available through June 14, 2007.

See you in Miami Beach!

Photo Information
Cover: Amy Biehl High School sophomores Catherine Olvera and Nate Maxson work on an in-class activity exploring fractional exponents.

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This Year in Horace

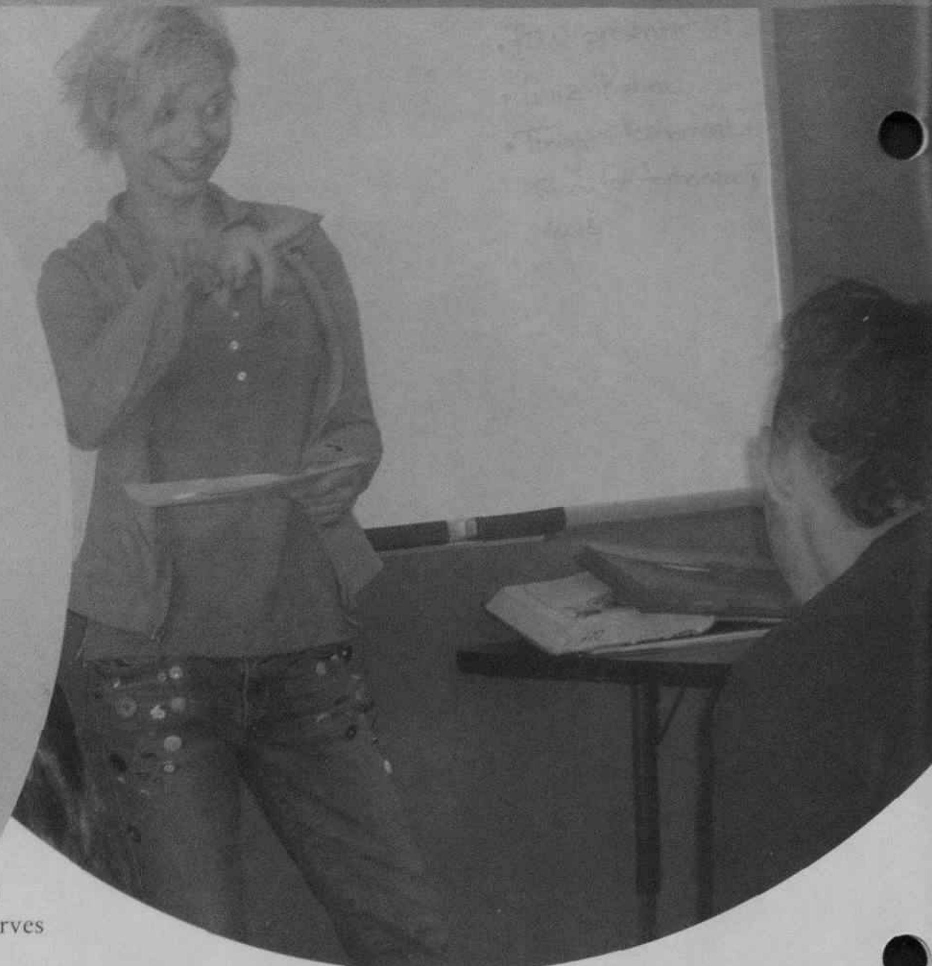
23.1: **Exhibitions: Demonstrations of Mastery in Essential Schools**
Horace focuses on the ways Essential schools use exhibitions, examining what is needed to implement an exhibition-based curriculum, analyzing exhibitions at various grade levels and within various disciplines, and discussing the impact of No Child Left Behind on exhibitions and vice versa.

23.2: **Essential Mathematics Education**
Essential school mathematics educators debate the advantages and challenges of responding to "less is more" and other CES Common Principles in mathematics, addressing what's happening now in Essential school math instruction.

23.3: **What's Essential about Elementary Schools?**
Horace looks at the latest thinking in the CES network about what defines CES elementary schools, inviting practitioners to discuss how elementary schools express the CES Common Principles.

23.4: **Beyond Reform: Transformations**
Horace explores how communities interrupt the status quo and create the conditions for transformed schools. How do transformed schools—and their larger environments—sustain and evolve as student-centered, collaborative, academically challenging and equitable places of learning?

The national office of the Coalition of Essential Schools gratefully acknowledges support from the following foundations:
The Bill and Melinda Gates Foundation and the Annenberg Foundation



Translating Success: How Careful Planning Within A Problems-Based Curriculum Can Prepare Students to Enter College-Level Math Classes

by Jessica Fillmore

Amy Biehl High School (ABHS) is a charter high school located in downtown Albuquerque that serves students from Albuquerque and the surrounding communities. There is no "typical" ABHS student; our student body is as rich and diverse as the city itself. Despite our school population's differences in skills, special needs, socioeconomic class, race, culture, and English proficiency, we have one common goal for all our students: that they are able to succeed in college. In fact, we ask students to prove that they are on their way to achieving this goal while they are still at our school through concurrent enrollment, requiring that they take and pass two college level courses in their senior year.

Given the overall mission of our school, it makes sense that the mission of the math team at ABHS is *for all students to succeed in a college level math class*, and this goal is the lens through which we examine everything that we do. The curriculum that we use in our efforts to achieve this goal is the Interactive Mathematics Program (IMP). IMP helps to prepare our students for success in college by instilling a deep understanding of the generation and application of various mathematical concepts. It fosters critical thinking and independence. It teaches students the value of questioning and collaborating with others when faced with a difficult problem. All of these skills are invaluable once a student is in the college classroom. But how does IMP prepare students for the tests that permit or deny access to college classrooms in the first place?

Curriculum Translation

Almost all colleges and universities use some sort of standardized test as a means of placing students in math classes. One local university uses an ACT math score, and another administers a placement test specific to that school. While we may not agree with the methods that colleges and universities use to place students into math classes, we do not feel empowered to change them anytime in the near future. Ensuring that students succeed in college math classes means making sure they get there in the first place. And that means preparing them for success on standardized exams. But how does the type of mathematics learned in a context-rich, problems-based curriculum translate to the isolated skills that a student will encounter on

Founded in 1999, Amy Biehl High School is located in downtown Albuquerque, New Mexico. A CES Mentor School, ABHS is a year-round school with 200 students in grades nine through twelve.

Sample Problems in Translation

Algebra

Solving basic equations is traditionally taught in the ninth grade in an Algebra I class. In IMP, it is covered at the beginning of Year 2 in the unit Solve It!

Solving Equations: IMP
There are 8 mystery bags of equal weight and 10 ounces of lead weights on one side of a pan balance, and 4 mystery bags and 30 ounces of lead weights on the other side. How much does each mystery bag weigh?

Solving Equations: Standardized Exam
Solve for x: $8x + 10 = 4x + 30$

Exponential Functions

Graphing exponential functions is sometimes covered in a ninth grade Algebra 1 course, but usually does not get taught intensively until Algebra 2. In IMP, graphing exponential functions gets covered at the end of Year 2 in the unit All About Alice.

Graphing Exponential Functions: IMP
Alice's height doubles for every ounce of cake that she eats. Find out what Alice's height is multiplied by when she eats 1, 2, 3, 4, 5, or 6 ounces of cake. Then make a graph of this information.

Graphing Exponential Functions: Standardized Exam
Graph the exponential function $y = 2x$

Trigonometric Ratios

Using trigonometric ratios to find missing side lengths in a right triangle is typically covered in a Geometry course, which most students take in their sophomore year. This topic is covered toward the end of Year 1 in IMP in the unit Shadows.

Using Trigonometric Ratios: IMP
Shredding Charlene is out surfing and catches the eye of her friend, Dave the Dude, who is standing at the top of a vertical cliff. The angle formed by Charlene's line of sight and the horizontal measures 28°. Charlene is 50 meters out from the bottom of the cliff. Charlene and Dave are both 1.7 meters tall. They are both 16 years old. The surfboard is level with the base of the cliff. How high is the cliff?

Using Trigonometric Ratios: Standardized Exam
A right triangle has a hypotenuse with a length of 12 feet and an acute angle that measures 27°. What is the length of the leg opposite the 27° angle?

such an exam? The key is in the idea of translation. It's as if our curriculum were written in one language and the tests in another. Our goal is for students to understand both languages.

Students can get all the skills they need to be successful college math students from the IMP curriculum. The problem comes when they have to take what they have learned in relation to a complex, weeks-long unit problem to the symbol-heavy and context-poor land of the standardized test. Though by now somewhat routine, it remains startling to watch a student struggle with a more "traditional-looking" problem just because of the way it appears, despite the fact that he or she has demonstrated to me an understanding of the concept repeatedly in the classroom. The task that faces us, then, is helping our students to be able to bridge the gap between the language of our curriculum and that different-looking "testing" language they will encounter as they transition from high school to college. We currently implement two strategies in an effort to help our students become better translators: one is the use of our own standardized exams, and the other is our outcomes-based grading system.

In-House Standardized Exams

One of the ways in which our team is working on bridging the gap is by giving our own standardized exam to all our students each year. These exams were created as a response to the need to better prepare students for the math portion of the ACT, which is the placement tool used by the local university that the majority of our graduates attend. Many of our students were not getting the required score of 22 that they needed on the ACT in order to be placed into College Algebra, so our team decided that we needed a structure in place that allowed us to collect data on our students' performance on a similar type of test over the course of the four years that they are with us. We created four in-house standardized exams - one for each grade level - and began administering them twice a year, once at the beginning of the year and once again at the year's conclusion. The questions for the test come directly from various high school

Continued on next page

graduation and college entrance exams to be sure that the testing language is authentic. Students' scores then become data that we can use to make decisions about our own curriculum and assessment.

But just how does a biannually administered, in-house standardized exam assist students in translating from the language of our curriculum to the language of the test? First of all, we are exposing our students to this "testing" type of language. It is our hope that seeing this language year after year in this type of setting will increase students' comfort and confidence with testing language. But this is more of a side benefit than our actual goal in doing this testing. The primary benefit of this process lies in the data that is generated by these tests and the ways in which the data informs how we plan.

Analyzing how students are doing on particular questions allows us to see where the gaps and holes are in our four-year curriculum. We even group the problems into various categories, and understanding how students do within those categories (such as *number sense* or *proportional reasoning*) allows us to see the areas in which our curriculum is weak as well as the areas in which it is strong. All of the skills necessary for success on standardized exams and within college classrooms are contained within the IMP curriculum; sometimes it's just a matter of figuring out where they are. Since the skills are embedded in a problems-based context, it takes a little work to identify correlations.

For example, in the Year 1 unit, The Pit and the Pendulum, students use the idea of curve fitting to determine if the prisoner's escape in Poe's famous story is realistic by predicting how long it would have taken Poe's pendulum to make 12 swings. While students are exposed to different curves in an investigative activity called "Graphing Free-for-All," it is up to the instructor to determine how formally these curves are treated and how much emphasis is placed on finding equations for various curves. The standardized exams allow us to create a solid structure that we confidently feel contains all the skills a student needs. And they allow us to be deliberate, making sure students know exactly what it is they are learning as well as exposing them to the way a skill might look were it taken out of the context of the unit problem they are attempting to solve.

Outcomes-Based Grading

Another useful tool in ensuring that students can translate their knowledge and skills on a standardized exam is the outcomes-based grading system. The outcomes system is a way of assessing student learning that the team has been developing over the past year. It involves the teacher pinpointing and naming the specific skills that are embedded in a given unit and then helping students to focus on and sharpen those skills throughout the unit. The teacher carefully investigates a unit before the students begin it, and then names the mathematical skills (usually eight or nine) that the student is required to master by the completion of the unit. The outcomes are shared with the students at the onset of the unit and serve as a map of sorts, helping them to navigate the sometimes muddy (in other words, not mathematically obvious) processes they perform over the course of a unit. Students are then held accountable for demonstrating to the teacher that they have mastered the outcomes in various settings, such as homework assignments, in-class assignments, and quizzes. Their work is judged to be either highly proficient, proficient, or not yet proficient, and they can continue to work on skills and readdress outcomes as necessary throughout a unit. Outcomes help students to name the skills that they are learning, and this naming helps to ensure that the skills are more easily translated beyond the context of the unit problem. Some examples of outcomes from the Year 2 unit, All About Alice, include solving problems involving exponents, graphing exponential functions, and understanding and using laws of exponents.

Students can see clearly the benefits of having an outcomes-based grading system. "With IMP, it's hard to know what exactly you're being taught because of how the assignments are set up," states Bethany Trujillo, a tenth grader at ABHS currently taking the second year of IMP. "Having outcomes it gives a clearer point of what we're doing, and also helps to show that we know and are capable of doing the work." Outcomes are solid and tangible; they are something that the student can go back to when they need to be reminded of the purpose of an assignment. Outcomes allow a student to connect the concepts they are learning with some sort of larger mathematical picture. Aine Brazil, a ninth grader taking IMP 2, says, "Before outcomes, I didn't know what was expected of me for each assignment, but now it is clear that each outcome is a skill that is important to learn to keep moving forward academically." Each outcome becomes a link in the chain, and seeing this allows students to transcend the contextual nature of IMP

Related Resource

For more on Amy Biehl High School, please read "Small School, Big Influence: Amy Biehl High School Tells Its Story" in *Horace* Volume 21, Issue 4, Fall 2005, www.essentialschools.org/cs/resources/view/ces_res/381 and connect their knowledge in new and innovative ways to spaces outside of the classroom. This, in turn, improves students' ability to retain and use skills in an unfamiliar setting, such as a standardized test.

In addition to helping students use skills in new ways, outcomes prepare students by naming skills and reinforcing those names so that students can recognize the name and then recall the skill in a new context. Because IMP is structured so differently from traditional math curricula, the book is not divided into discrete processes with the name of a particular process in bold letters at the top of the page. Math in context is rich and powerful in that it gives students an understanding of the need for the mathematical concepts they are learning. But sometimes the names can get lost in the context.

For example, an IMP student might feel quite comfortable finding the area of a rectangular lot with a length of $x + 3$ and a width of $x + 4$, but might have no idea what was being asked of him or her if a problem said "multiply two binomials." Outcomes allow students to become familiar with the names of concepts and skills and thus increase the likelihood that a student will accurately interpret the directions of a problem on a standardized exam. Students are thus equipped to more accurately and easily translate from a contextual setting to more formal mathematical language and back again.

Opening the Gates

The ABHS math team does not propose that simply teaching to the test prepares students for success in college, though these translation skills have value beyond the various gatekeeping standardized tests that students will encounter as they move from ABHS to their next educational steps. Preparing students for the more formal language of standardized tests also prepares them for success in the college classroom where that more formal language is typically employed. But the unfortunate reality is that students have to have access to a class before they can succeed in the class. It is our sincerest hope that someday a new system will come into place that determines the classes in which students belong based on a careful examination into the mathematical background of each student. But in the meantime, we believe that our mission is best achieved by working to prepare students to succeed on these tests and pass through these gateways. We haven't changed what we teach. We've just been careful to expose students to what the skills they are already learning might look like in a different context. We believe that familiarizing students with the more formal and rigorous language of mathematics that they will encounter in their post-secondary studies allows them to translate the deep and useful learning that occurs in a curriculum like IMP to this new setting.

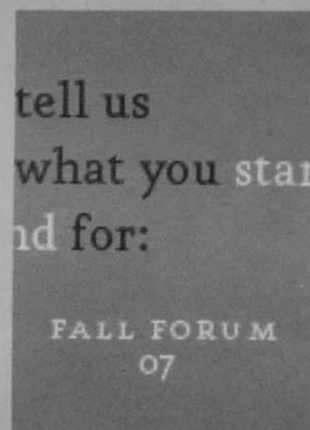
Jessica Fillmore has been teaching math for three years. She has been teaching the Interactive Mathematics Program at Amy Biehl High School for the past two years.

Get Ready for Fall Forum 2007!

A Principled Stand
November 8-10
Denver, Colorado

Get ready to join in this year's conversation among friends at the Adam's Mark Hotel in Denver. Registration for Fall Forum opens on our website, www.essentialschools.org, in late August.

Fall Forum is the centerpiece of the CES network's year, attracting thousands of educators, students, family members, administrators, policy-makers, and researchers committed to using the CES Common Principles to create equitable, personalized, and academically vibrant schools for all children.



This year's theme is A Principled Stand. To get ready, we are collecting comments from around the CES network about what we stand for. We plan to use these brief comments in conference materials, on our website in the coming months, and at the conference itself.

We want to hear from you! Please visit www.essentialschools.org/fallforum.html to tell us about the relationships, pedagogies, school structures, and educational policies that you stand for.

We're looking forward to seeing many new faces and lots of old friends in Denver in November!

What if Less Is Just Less?

The Role of Depth Over Breadth in The Secondary Mathematics Curriculum

by Roser Giné and Diane Kruse

One of the most challenging Common Principles for mathematics educators in Essential schools to implement is "less is more." We are acutely aware of the role of mathematics performance as a gatekeeper; college entrance and placement exams rely heavily on math scores, and the current emphasis on high-stakes testing makes passing math exams a high school graduation requirement in many states. Once our students get to college, they may find themselves paying for math classes for which they earn no credit; many colleges will not give credit for any math class below college algebra (precalculus).

In addition to these immediate obstacles for our students, problems and challenges facing mathematics education include: low mathematics scores of American students in comparison to students from other industrialized nations, decreased student enrollment in undergraduate and graduate mathematics programs, as well as lack of mathematical competence in today's workforce. Although the need for mathematics within science and technology fields is significant, its role goes beyond career preparation; mathematics reasoning is an indispensable tool for informed participation in a democracy. Information and knowledge of our world is increasingly understood and disseminated through the examination of patterns and trends; consequently, decision-making within our society necessitates the individual's ability to sort through relevant information and synthesize facts that affect a particular issue.

At the secondary school level, the achievement gap in mathematics persists, even though federal and state mandates have increased the number of mathematics course requirements for all students. Research indicates that African American, Latino, and Native American students continue to score lowest on standardized assessments. Few of these students then continue to study mathematics beyond lower level courses in high school. This creates a situation for student populations most at risk and, as stated in the 1989 National Research Council's Report, "No one – not educators, mathematicians, or researchers – knows how to reverse a consistent early pattern of low achievement and failure. Repetition rarely works; more often than not, it simply reinforces previous failure"

Teachers and schools who want the best for their students in this context are rightfully pushing for more mathematics instruction. This may seem to be at odds with "less is more," since the first thing many people assume they must do to live out this principle is to start cutting content – and everything looks too important to omit.

In our practice as math teachers and curriculum designers in several different Coalition schools, we have come to see "less is more" in a different light. Rather than a command to cut back, we see this principle as an invitation to consider the role of mathematics education through a different lens, with the following question guiding our work: "What is essential for students to take away from their high school mathematics education?"

When we begin to design programs around the larger understandings and habits of mind that answer this question, we build mathematics programs with a coherence and vision that feel like "less" to students, as they focus on bigger questions that they investigate in depth. At the same time, that laundry list of content that feels so important to cover still exists, but within a structure that allows students to understand and retain what they have learned.

We focus first on uncovering what it really means to be quantitatively literate. The following characterization expressed by Alan Schoenfeld provides us with a conceptualization of what we want for our students and a starting point for curriculum design and implementation:

> "Quantitative literacy is the ability to interpret the vast amounts of quantitative data [one] encounters on a daily basis, and of making balanced judgments on the basis of those interpretations. Quantitatively literate people are flexible thinkers with a broad repertoire of techniques and perspectives for dealing with novel problems and situations. They are analytical, both in thinking issues through themselves and analyzing arguments put forth by others."

This definition is congruent with recent reform efforts that have focused on helping students learn to think like mathematicians within their classroom settings. Mathematics education is no longer skill development through routine tasks; rather, it is an effort to present students with complex situations where there is no set solution, and the process of analysis, or breaking apart a phenomenon to understand its components and their effects on one another, takes precedence. Our goal is to help students become well-versed in mathematical language and proficient in symbolic manipulation so that they internalize the tools of mathematics; in turn, we can provide them with messy problems like those mathematicians encounter, not just the formal structures through which mathematicians present their final results. Our task thus has shifted to one that demands students to take ownership of their learning through the exploration of complex problem situations, while teachers provide necessary guidance for students to develop and access relevant mathematical knowledge.

We have attempted to implement "less is more" in a variety of educational contexts. The schools we have been part of include a pilot school in Boston, New Mission High School (New Mission), a charter school in Fitchburg, North Central Charter Essential School (NCCES), and a charter school in Devens, Francis W. Parker Charter Essential School (Parker). The three schools are members of the Coalition of Essential Schools and were founded as such; although New Mission and Parker share more than ten years of existence, they differ in the population each serves. New Mission is considered an inner-city school, while Parker mostly serves suburban residents. NCCES, the newest of the three schools, is an urban school in its fifth year of operation. Although the challenges due to demographics guided our work in each of the three schools, it is beyond the scope of this article to address these in detail.

The Programs

All three of the programs we describe here have used the Massachusetts state curriculum frameworks to inform the course content. Students in these schools must pass the MCAS, the state math exam, in order to earn a high school diploma, and in the case of the two charter schools, their very existence depends on regular charter renewals that closely examine the students' academic performance.

The educators designing these programs have looked to find connections between topics that have been separate, and integrated them through the use of different unifying concepts, expressed through essential questions that capture the thematic focus of the units. The programs spiral so that students see different concepts several times in increasing depth. And while all teachers in these programs model some important mathematical procedures and techniques through direct instruction and practice, they consistently make space in their courses for deeper explorations. The curriculum in these schools is not just a list of things to know, but demands that students use, explore, play around, discover, make connections, and problem solve.

Related Resource

For more from Roser Giné on her work at New Mission High School, read "What Is Essential in a High School Mathematics Curriculum Framework?" from *Horace* Volume 20, Issue 2, Winter 2004.

New Mission High School: Mathematical Elements

The mathematics curriculum framework at New Mission High School (where author Roser Giné taught from 1998 to 2004) evolved from leader- and

Continued on next page